

Techniques of Water-Resources Investigations
of the United States Geological Survey

**MEASUREMENT OF DISCHARGE
USING TRACERS**

Book 3
CHAPTER A16





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of the United States Geological Survey

Chapter A16

MEASUREMENT OF DISCHARGE
USING TRACERS

By Frederick A. Kilpatrick and Ernest D. Cobb

Book 3

APPLICATIONS OF HYDRAULICS

DEPARTMENT OF THE INTERIOR

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UNIT CONVERSION

<i>Multiply inch-pound unit</i>	<i>By</i>	<i>To obtain SI unit</i>
cubic foot (ft ³)	28.320	cubic meter (m ³)
cubic foot per second (ft ³ /s)	0.028	cubic meter per second (m ³ /s)
degrees Fahrenheit (°F) minus 32°	0.556	degrees Celsius (°C)
foot (ft)	3.048	meter (m)
foot per second (ft/s)	0.305	meter per second (m/s)
gallon (gal)	3.785	liter (L)
gallon (gal)	3785	milliliter (mL)
inch (in)	25.4	millimeter (mm)
mile (mi)	1.609	kilometer (km)
pound (lb)	453.6	gram (g)
square foot per second (ft ² /s)	9.290	square meter per second (m ² /s)

SYMBOLS, DEFINITIONS, AND UNITS

<u>Symbol</u>	<u>Definition</u>	<u>Unit</u>
A _c	Area under the time-concentration curve	(μg/L)(min)
B	Average width of stream	ft
c	Plateau concentration	μg/L
c _i	The concentrations at points i across a section	μg/L
\bar{c}	Weighted or average plateau concentration	μg/L
C	Concentration of injected solution	μg/L
C _a	Average concentration for an entire tracer cloud when sampled at a constant rate over a time period, ΔT _c	μg/L
C _i	Initial concentration	μg/L
C _n	New concentration after a dilution	μg/L
C _p	Peak concentration	μg/L
C _s	Concentration of stock solution, generally as obtained from the manufacturer	μg/L
C _{1,2,3,4}	Concentrations resulting from serial dilutions 1 through 2, 3, and 4 dilution steps	μg/L
C _f	Final standard concentration	μg/L
d	Mean depth of stream	ft
D	Diameter of orifice	in
D _T	Total dilution factor; the product of a series of dilutions, D ₁ ×D ₂ ×D ₃ , and so forth	---
D _{1,2,3,4}	Dilutions for each step, 1, 2, 3, and 4 of a serial dilution, respectively	---
D _C	Dilution factor to reduce C _s to C	---
E _z	Transverse mixing coefficient	ft ² /s
g	Acceleration of gravity	ft/s ²
h	Head on orifice	in
K	Mixing length coefficient	ft ²
k	Fluorometer scale factor	---
L	Length of measurement reach	ft
L _m	Mixing length	ft
L _o	Channel length required for optimum mixing; usually corresponds to about 95 percent mixing	ft
M	Mass of tracer injected	g
M _{1,2...n}	Series of slug injections of equal mass	g
P _m	Percentage of mixing	---
q	Rate of constant tracer injection	mL/min
Q	Total stream discharge	ft ³ /s
Q _{a,b,c...n}	Portions of the total stream discharge, Q	ft ³ /s
Q _i	Discharges applicable to the points, i	ft ³ /s
r	Fluorometer dial reading for individual stream sample	---
\bar{r}	Weighted or average fluorometer dial reading for stream samples	---

R	Fluorometer dial reading for a standard prepared from the injection solution or prepared from stock solution used in preparing the injection solution	---
s	Water-surface slope	ft/ft
S_G	Specific gravity	---
ΔT_c	Period during which dye cloud is sampled at a constant rate	min
t_c	Lapsed time between centroids of time concentration curves	s
T_d	Duration in time for tracer cloud to pass any one point in section	min
T_D	Duration in time required for entire tracer cloud to pass a section	min
T_e	Elapsed time to leading edge of dye cloud	min
T_f	Elapsed time to trailing edge of dye cloud	min
T_p	Elapsed time to maximum concentration of dye cloud	min
v	Mean stream velocity	ft/s
v^*	Shear velocity	ft/s
V_d	Volume of dye solution	mL
V_I	Volume of dye solution injected into stream	mL
V_s	Volume of stock dye solution	L
V_w	Volume of diluent, usually distilled water	mL
V	Volume of conduit	ft ³
W_d	Weight of dye	g
W_s	Weight of stock dye solution	g

EQUATIONS

For Slug-Injection Method

$$(1) \quad Q = \frac{M}{A_c} \text{ (without unit conversion factor)}$$

$$(3) \quad Q = 5.89 \times 10^{-7} \frac{S_G V_I C}{A_c}$$

$$(8) \quad Q = 5.89 \times 10^{-7} \frac{S_G V_I C}{\Delta T_c C_a}$$

For Constant-Rate Injection Method

$$(2) \quad qC = Qc \text{ (without unit conversion factor)}$$

$$(9) \quad Q = 5.89 \times 10^{-7} q \frac{C}{\bar{c}}$$

$$(10) \quad Q = 5.89 \times 10^{-7} q \left[\frac{R}{\bar{F}} \times \frac{D_C}{D_T} \right]$$

$$(11) \quad Q = 5.89 \times 10^{-7} q \left[\frac{R}{\bar{F}} \times \frac{1}{S_G D_T} \right]$$

For Mixing Length Determinations

$$(4) \quad L_0 = 0.1 \frac{vB^2}{E_z}$$

$$(5) \quad L_0 = 0.088 \frac{vB^2}{d^{3/2} g^{1/2}}$$

$$(12) \quad L_m = K \frac{vB^2}{E_z}$$

For Volume of Rhodamine WT Dye

$$(6) \quad V_s = 3.79 \times 10^{-5} \frac{QL}{v} C_p$$

For Serial Dilution Determinations

$$(7) \quad C_n = C_i \left[\frac{W_d}{V_w + V_d} \right] = C_i S_G \left[\frac{V_d}{V_w + V_d} \right]$$

For Computing Percentage Mixing

$$(14) \quad P_m = 100 - \frac{50}{\bar{c}Q} \sum_{i=1}^N |c_i - \bar{c}| Q_i$$

For Tracer Velocity Discharge Measurements

$$(13) \quad Q = \frac{V}{t_c}$$

MEASUREMENT OF DISCHARGE USING TRACERS

By Frederick A. Kilpatrick and Ernest D. Cobb

Abstract

The development of fluorescent dyes and fluorometers that can measure these dyes at very low concentrations has made dye-dilution methods practical for measuring discharge. These methods are particularly useful for determining discharge under certain flow conditions that are unfavorable for current meter measurements. These include small streams, canals, and pipes where

1. Turbulence is excessive for current-meter measurement but conducive to good mixing.
2. Moving rocks and debris may damage instruments placed in the flow.
3. Cross-sectional areas or velocities are indeterminate or changing.
4. The flow is unsteady, such as the flow that exists with storm-runoff events on small streams and urban storm-sewer systems.
5. The flow is physically inaccessible or unsafe.

From a practical standpoint, such methods are limited primarily to small streams, because of the excessively long channel-mixing lengths required for larger streams. Very good accuracy can be obtained provided that

1. Adequate mixing length and time are allowed.
2. Careful field and laboratory techniques are used.
3. Dye losses are not significant.

This manual describes the slug-injection and constant-rate injection methods of performing tracer-dilution measurements. Emphasis is on the use of fluorescent dyes as tracers and the equipment, field methods, and laboratory procedures for performing such measurements. The tracer-velocity method is also briefly discussed.

Introduction

Tracer gaging may be divided into two categories: the tracer-dilution method and the tracer-velocity method. The latter is commonly known as the salt-velocity method (Aastad and Reinhardt, 1954), although any tracer may be used. It has the advantage of simple injection, requires a relatively small amount of tracer, and is not critically dependent on the conservativeness of the tracer. Its chief disadvantages are the requirements that the channel reach be uniform and that the channel-flow geometry be defined exactly; it also entails extensive sampling. This method, although not frequently used, can be very accurate in canals and pipes

where the cross-sectional area is uniform and accurately measurable. It is discussed in a later section.

Measurement of stream discharge by dilution methods depends on the determination of the degree of dilution of an added tracer solution by the flowing water. Dilution methods of measuring discharge have been known since at least 1863 (Spencer and Tudhope, 1958, p. 129). Until recently, chemical salts (Ostrem, 1964) were generally used as the tracer injected into the stream. Radioactive tracers have been used successfully, but handling problems have limited widespread use (Schuster and Hansen, 1968). The development of stable fluorescent dyes and fluorometers that can measure them at very low concentrations has greatly enhanced the use of dilution methods (Morgan and others, 1977, and Kilpatrick, 1968, 1969). Hence, this manual addresses the use of fluorescent dyes as the tracer to be used, although the principles discussed apply to any tracer.

Dilution methods are useful under the following flow conditions:

1. Where it is difficult or impossible to use a current meter due to high velocities, turbulence, or debris.
2. Where, for physical reasons, the flow is inaccessible to a current meter or other measuring device.
3. Where, for some conditions, the rate of change of flow is such that the time to make a current-meter measurement is excessive.
4. Where the cross-sectional area cannot be accurately measured as part of the discharge measurement or is changing during the measurement.

Typical examples of situations where the dilution methods might be used are turbulent mountain streams, pipes, canals, sewers, ice-covered streams, and sand-channel streams. The purpose of this manual is to describe the dyes, equipment, and methods used to measure discharge by the dilution method using both the constant-rate injection and the slug-injection technique.

Note that this manual goes beyond what is required to make a simple dilution-type discharge measurement and that the user need read through only the sections on Theory, Slug-Injection Method, and Constant-Rate Injection Method before attempting such measurements. Also bear in mind that while many of the same techniques as those used in time-of-travel studies are used in making dye-dilution discharge measurements, a higher level of accuracy is desired, and therefore greater care in performance is required. The user is urged to perform a dye-dilution discharge measurement on an ordinary small stream before attempting more difficult measurements. The stream chosen for a trial should be one where a good current-meter or weir measurement can be made concurrently to provide verification.

The fluorometric techniques described in this report are oriented toward use of the Turner Model 111 and Turner Designs Model 10 fluorimeters because those instruments are in general use in the United States Geological Survey; however, other brands of fluorimeters may be used. Additional information about tracers and equipment is given in the manual by Wilson and others (1984). The methods and techniques presented in this manual are based on experimental studies by F. A. Kilpatrick and E. D. Cobb, and by B. J. Frederick, J. F. Wilson, Jr., J. F. Bailey, and numerous others.

Theory

Measurement of the degree of dilution of a known quantity of tracer after its mixing in a flowing stream of water is the basis of dilution gaging. There are two main approaches: (1) the slug injection of a known amount of tracer into the flow, which requires that the dilution of the tracer be accounted for by the complete measurement of its mass downstream; for this reason, it is sometimes referred to as the total recovery method; and (2) the constant-rate injection of a tracer solution into the flow, which requires only the measurement of the plateau level of concentration that results downstream after equilibrium has been reached. The principles are simple, yet their successful application in streams, canals, pipes,

and elsewhere requires a good understanding of the dispersion process. No elaborate theoretical treatment is used in explaining these processes; however, the reader is urged to heed the following principles, for they can eliminate many of the problems others have had in performing such measurements and are helpful in understanding the application of tracers in hydrologic studies in general.

Slug injection

The slug injection of a tracer into a flowing stream is the simplest of all methods, from the standpoint of equipment needs. Where radioactive tracers are employed, handling problems make slug injection virtually the only feasible method. Figure 1 shows the resulting response curves at different distances downstream that may result from a single midchannel slug injection of tracer. These response curves are time-concentration curves familiar to many who have been involved in time-of-travel studies; however, stream reaches used for a dilution-discharge measurement are relatively short, and the duration of passage of the entire cloud, T_D , rarely exceeds 1 hour. Large distances such as those in time-of-travel studies may result in significant loss of tracer and, hence, inaccurate measurements of discharge. Significant losses of tracer do not ordinarily occur in the short stream reaches used for dilution-discharge measurements, although the possibility must always be kept in mind.

The discharge as measured by the slug-injection tracer-dilution technique is

$$Q = \frac{M}{A_c} \quad (1)$$

where Q is the volume rate of flow of the stream;

M is the mass of tracer injected; and
 A_c is the area under the response curve obtained after adequate mixing of the tracer in the flow.

¹For clarity, units are ignored here; a constant is required to yield a dimensionally correct equation for the units used in this report.

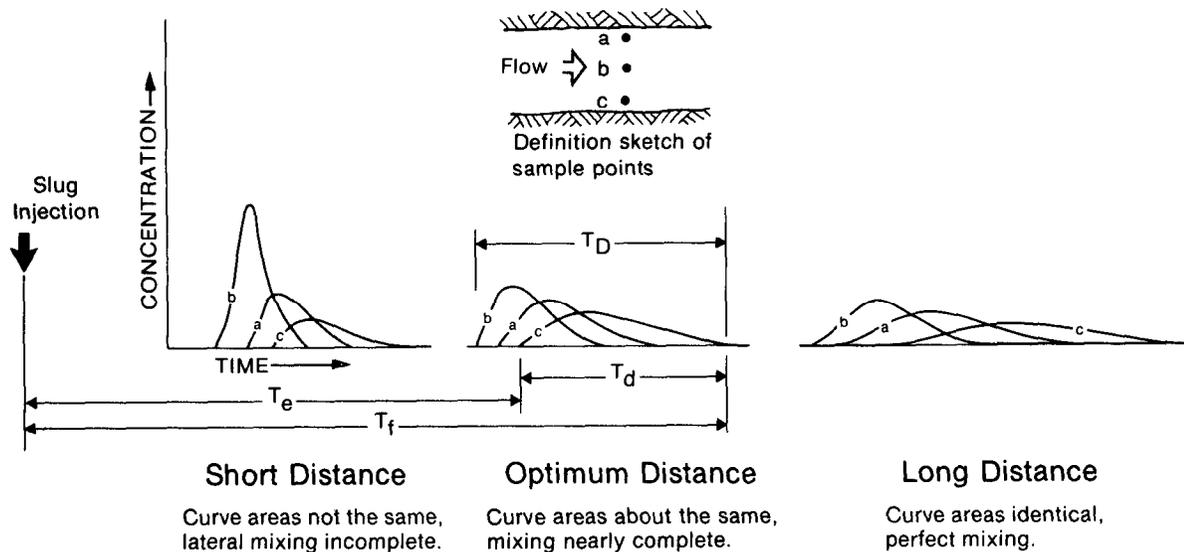


Figure 1.—Typical response curves observed laterally and at different distances downstream from a slug injection of a tracer in the center of a stream.

Most important, therefore, is a measurement of the response curve far enough downstream that mixing is almost complete in a cross section.

At short distances downstream from a slug injection, the tracer is not fully mixed in the total flow of the stream, being more in the center than along the banks (see fig. 1). Furthermore, the response curve measured in the center may be much shorter in duration, T_a , than for those along the banks. This is a common occurrence, because flow along the banks is usually slower and the channel banks tend to slow and elongate the tracer cloud. At such a short distance, an accurate measurement of discharge by dilution cannot be made by ordinary methods. When uniform mixing is reached, the areas under the time-concentration curves are essentially the same regardless of shape; at too short a distance, they are not.

From a practical standpoint, complete mixing does not need to be attained. A good dilution-discharge measurement can be made at what is defined here as an optimum distance, L_o , downstream.² The distance is optimum be-

cause T_D is not too long, and thus sampling of the complete response curves at several points laterally across a section is feasible. Note that the peaks of the response curves are not the same, and that their lengths, or durations, and arrival and departure times are different. Nevertheless, the areas under the individual response curves are nearly the same, which indicates good mixing and allows a good dilution-discharge measurement.

If the response to the tracer slug is measured farther downstream, mixing will be nearly perfect, and the individual time-concentration curves will be nearly identical in area; therefore, a very accurate measurement of discharge may be obtained. That is true, however, only if sampling has been performed for a sufficient time, particularly of the drawn-out tails of the clouds, and if loss of tracer has not occurred because of excessive time of exposure. Frequently, failure to measure the low-level concentrations of the trailing edge of the tracer is misinterpreted as a loss of tracer. It is not a real loss, but merely a sampling or measurement failure.

The foregoing discussion shows that predicting the length of channel for optimum mixing is not a simple task. Formulas to aid in estimating mixing lengths and approaches to reducing the effective reach lengths needed are discussed subsequently.

²The optimum distance is usually where mixing is about 95 percent complete. The concept of degree of mixing and how it is computed is discussed in appendix A.

Constant-rate injection

A continuous, constant-rate injection of tracer may be simulated from the response curve of a slug injection using the superposition principle (Yotsukura and Kilpatrick, 1973). This can best be understood by using any one of the slug-response curves in figure 1 to simulate the response of a continuous injection at the same location. In figure 2, the solid response curve is due to the slug injection, M_1 ; T_e and T_f are the elapsed times to the arrival of the leading edge and trailing edge of the response curve to M_1 . Assuming the streamflow is steady, continuing to inject a series of tracer slugs of equal amounts, M_2, M_3, \dots, M_n , at uniform time intervals (a constant-rate injection), would yield a series of identical response curves. Of course, if the same soluble tracer were continuously injected, the individual response curves would not be distinguishable and there would be an ever-increasing buildup of concentration with time until T_f was reached. In effect, the superimposed slug-response curves are being added as they overlap. At time T_f , corresponding to when the trailing edge of the first slug would have passed the point of observation, a plateau of constant concentration is first reached at that point in the channel. At that time, for a constant injection, an equilibrium condition is reached. Continued injection after T_f would re-

sult in a plateau of constant concentration at that point, so long as the stream discharge and rate of tracer injection did not vary.

The results illustrated in figure 2 are shown also in figure 3 for the same three distances and the three lateral locations used in the example of figure 1. The same slug-response curves used in figure 1 are used in figure 3 to simulate the responses to a continuous injection of tracer as obtained by superimposing the uniformly repeated slug-response curves.

Figure 3 shows that for the short distance, plateaus of different concentrations are obtained laterally. The degree of mixing is poor and identical for the continuous injection and the slug injection.

At the optimum distance, the plateaus laterally tend to converge to the same concentration and reflect dilution by the stream discharge, but their concentrations are not identical; nevertheless, mixing is sufficient to produce a good tracer dilution-discharge measurement. Note that tracer would have to be injected continuously for at least a time, T_D , to first reach a completely stabilized plateau for all points across the channel at that distance. The minimum period corresponds to the difference in time for the earliest arriving tracer (center or point b) and the latest departing tracer (side or point C). Of course, it would be necessary to inject tracer for a time slightly greater than T_D

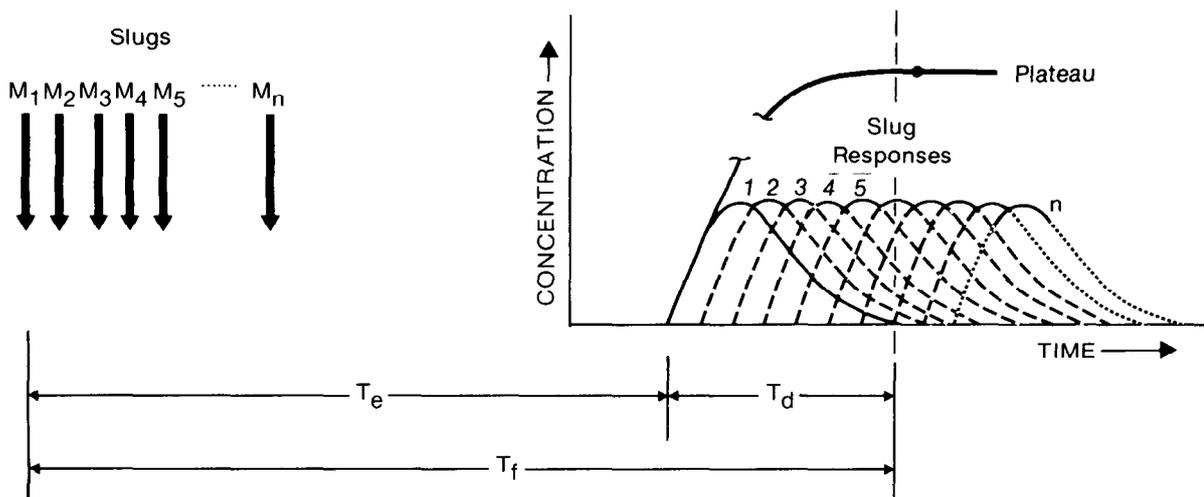
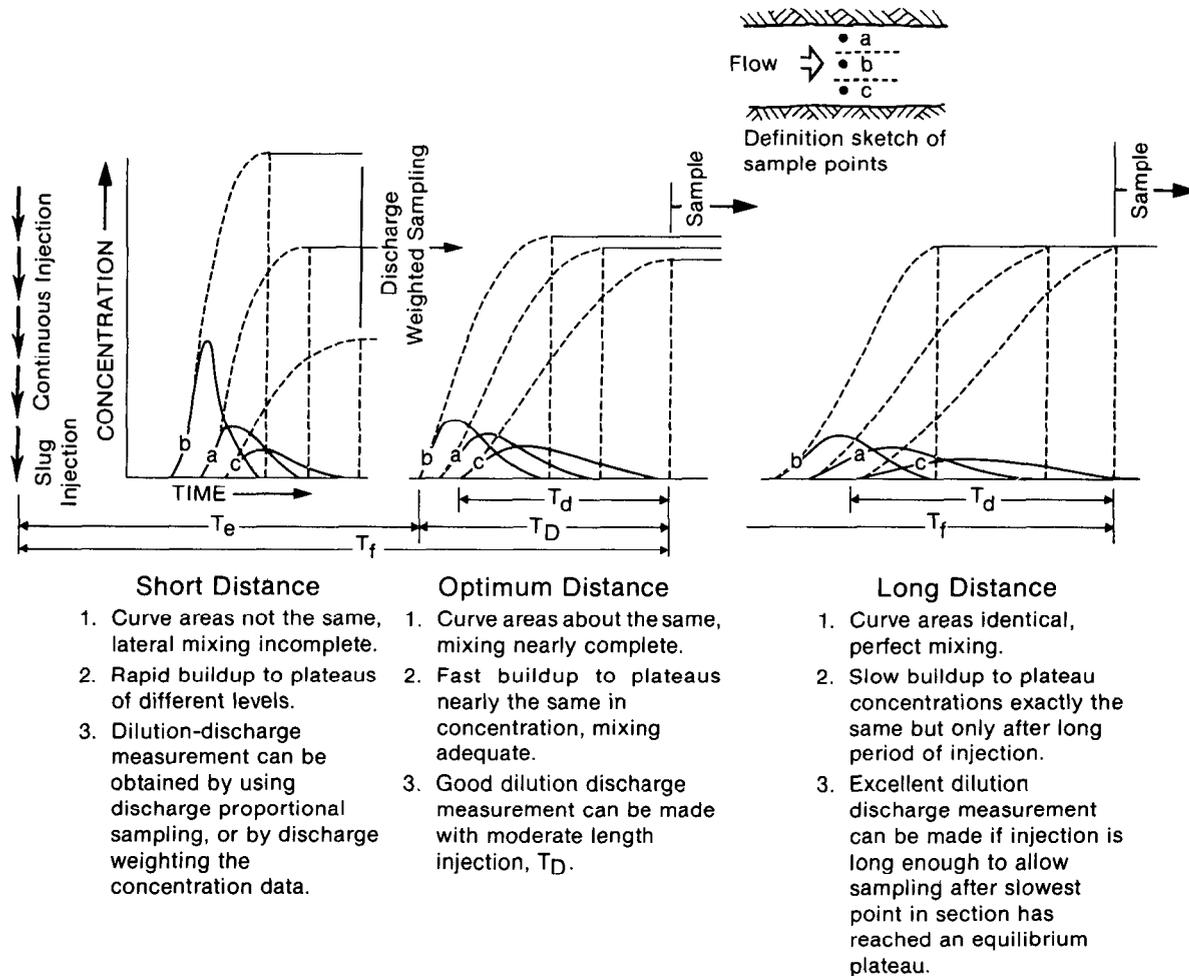


Figure 2.—Superposition of slug-response curves to simulate constant-injection buildup to a plateau at one location in a stream section.



GENERAL: Minimum length of constant injection required, T_D , is equal to longest time slug dye cloud would be present at any location in the stream. Danger is in not injecting long enough and not sampling late enough, samples taken prior to T_f would give impression mixing was poor and (or) yield data suggesting, incorrectly, a tracer loss

Figure 3.—Time-concentration curves for slug and simulated constant injections observed at three points laterally across the channel at three different distances downstream from the injection point.

(shown only for the optimum distance) to allow for sampling the fully developed plateau. At a long distance (fig. 3), the resulting plateau concentrations are virtually identical if the constant injection is made over sufficient time and if sufficient time is allowed for buildup across the entire channel before sampling. Depending on the nature of the channel and the mixing distance selected, the injection duration and the lapsed time necessary before sampling the correct plateau concentration may be greater than anticipated.

Unlike the slug-injection method, the entire response curves need not be measured, only the plateau concentrations. Once equilibrium plateau conditions have been reached, the conservation-of-mass principle and the continuity equation

$$qC = Qc \quad (2)$$

³For clarity, units are ignored here; a constant is required to yield a dimensionally correct equation for the units used in this report.

apply, whereby the amount of tracer being injected, qC , equals that passing the sampling section, Qc ,

where q is the rate of tracer injection and is assumed to be very small relative to Q ;

C is the concentration of the tracer being injected; and

c is the resulting plateau concentration after dilution by Q .

Certain conclusions can be drawn from examination of figures 1, 2, and 3 for a given stream and flow:

1. Sampling of the response curves from a slug injection must be for a period of time, T_D , and until time T_f ; T_f is the earliest time at which the plateau resulting from a constant injection can be sampled. Thus, the effective elapsed time required to make a discharge measurement is essentially the same by either method.
2. The duration of a continuous injection needed to reach equilibrium plateau concentrations can be determined accurately by examination of the slug-response curves for a given site. The injection time must be at least equal to the time at which tracer is last present in the stream minus the earliest time of arrival of the leading edge of the tracer for that section, T_D . For practical purposes, the injection time must be slightly longer than T_D to allow for sampling.
3. For the constant-injection method, plateaus develop earlier in the main flow than they do close to the streambanks, where the flow is slower.

Slug-Injection Method

Overview

The slug-injection method is sometimes preferred because of the simplicity of injection and because less tracer is needed. Equation 1 may be written as

$$Q = 5.89 \times 10^{-7} \frac{S_G V_I C}{A_c} \quad (3)$$

where Q is the discharge of the stream, in cubic feet per second;

V_I is the volume of concentrated dye solution injected into the stream, in milliliters;

C is the concentration of the dye solution injected into the stream, in micrograms per liter;

A_c is the area under the time-concentration curve, in units of minutes time micrograms per liter; and

S_G is the specific gravity of the injected solution.

Successful application of equation 3 requires that the mass of tracer injected be fully recovered at the sampling point. This recovery consists in multiplying the area of the response curve or time-concentration curve, A_c , by the discharge, Q . Thus it is vital that A_c accurately represent the dilution of the injected tracer. To recover the injected tracer totally, therefore, requires complete measurement of the time-concentration curve at each of several locations laterally, such as at a, b, and c as depicted in figure 1. The necessity for extensive sampling is the reason this method has found less favor in the United States than the constant-injection method. Nevertheless, the method may be used where a constant-rate injection apparatus is not available or its use is not feasible. Modification of the method to decrease the number of samples required can make it more attractive. Such modification is discussed after presentation of the basic method.

Preparation

Reach selection

The reach where the discharge measurement is to be made should be close to the stream-gaging station or other point of interest. There should not be inflow or outflow within the measurement reach or between the measurement reach and the gage.

The measurement reach should contain a minimum of dead-water areas. Such areas can greatly extend the time required for the tracer to pass a downstream section and, hence, the measurement time. Also, extensive deadwater

Table 1.—Values of the transverse mixing coefficient, E_z , for selected average flow depths and slopes.[Note: $E_z=1.13d^{3/2}s^{1/2}$]

Depth, d (ft)	Slope, s, (ft/ft)				
	0.001	0.005	0.010	0.050	0.100
1.0	0.04	0.08	0.1	0.3	0.4
2.0	.1	.2	.3	.7	1.0
3.0	.2	.4	.6	1.3	1.9
4.0	.3	.6	.9	2.0	2.9
5.0	.4	.9	1.3	2.8	4.0

areas will elongate the trailing edge of the tracer cloud and result in long-duration, low-concentration tails that it is difficult to define accurately.

Rapid lateral (cross-channel) mixing is desired in the stream reach to reduce measurement time and length of reach. A stream with good mixing characteristics will be narrow and deep. Mixing will usually take place rapidly in pool-and-riffle streams--the pools generally being more effective for lateral mixing than the riffles. The volume of the pools should not be too great relative to the volume of flow; otherwise it will take excessive time for the tracer to move through them. Contractions and bends within the reach may help speed up the mixing process. Lateral mixing in shallow whitewater streams is poor, and its mixing effectiveness is apt to be deceiving; depth is needed for good lateral mixing. For the same reason, braided reaches should be avoided. Small islands generally will not significantly slow the mixing process, provided that flow characteristics are similar on both sides of an island.

The optimum distance required for adequate lateral mixing of a tracer injected in the middle of the streamflow may be estimated by the following equation (adapted from Yotsukura and Cobb, 1972, eq. 29, and Fischer and others, 1979, eqs. 5 and 10):

$$L_0 = 0.1 \frac{vB^2}{E_z} \quad (4)$$

where E_z is the transverse mixing coefficient (table 1) in feet squared per second, given by

$$E_z = 0.2 d v_*$$

where v_* is the shear velocity, in feet per second, given by $v_* = (g d s)^{1/2}$;

L_0 is the distance required for optimum mixing, in feet;

d is the mean depth of the stream, in feet;

g is the acceleration of gravity (32.17 ft/s²);

s is the water-surface slope, in feet per foot;

v is the mean stream velocity, in feet per second; and

B is the average stream width, in feet.

The transverse mixing coefficient may therefore be expressed as

$$E_z = 1.13 d^{3/2} s^{1/2}$$

Hence, the optimum mixing length may also be expressed as

$$L_0 = 0.088 \frac{vB^2}{d^{3/2}s^{1/2}} \quad (5)$$

Table 1 provides values of E_z for selected depths and slopes to aid in estimating the optimum mixing length from equation 4.

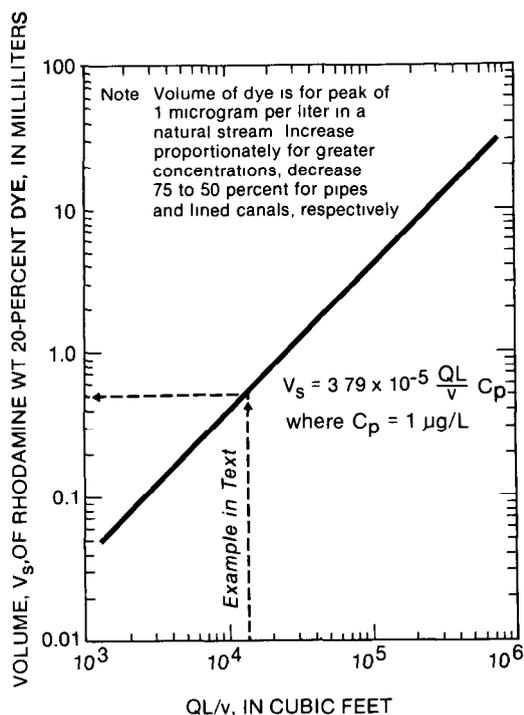


Figure 4.—Quantity of rhodamine WT 20-percent dye required for slug injection to produce a peak concentration of 1 microgram per liter at a distance downstream, L , at a mean velocity, v , and with a discharge, Q , in the reach.

Estimation of needed tracer amounts

The volume of tracer required for a slug-injection discharge measurement is a function of the stream discharge, the measurement-reach length, the stream velocity, and the peak concentration to be achieved at the sampling site. Henceforth, this manual will describe the use of rhodamine WT and pontacyl pink dye tracers, particularly the former. The following easy-to-use empirical equation for estimating the dosage requirements of rhodamine WT 20-percent dye for natural streams was developed by Kilpatrick (1970):

$$V_s = 3.79 \times 10^{-5} \frac{QL}{v} C_p \quad (6)$$

where C_p is the peak concentration at the sampling site, in micrograms per liter;

L is the length of the measurement reach, in feet;

Q is the stream discharge, in cubic feet per second;

V_s is the volume of rhodamine WT 20-percent dye, in milliliters; and

v is the mean-stream velocity, in feet per second.

A plot of equation 6 for a value of $C_p = 1.0 \mu\text{g/L}$ is shown in figure 4. For most discharge measurements, a peak concentration of 10 to 20 $\mu\text{g/L}$ is recommended. Experience indicates that in lined channels and smooth pipes flowing full, the dosages may be reduced to one half to one fourth, respectively, of the amount needed in a natural stream, as defined by equation 6. In such cases, longitudinal dispersion is less and, hence, less dye is required to produce a desired peak.

Pontacyl pink dye comes as a 100-percent pure powder. It compares favorably with rhodamine WT as a water tracer and can be analyzed on most fluorometers using the same lamps and filters as for rhodamine WT. The weight in grams of pontacyl pink needed for a slug injection can be determined from equation 6 or figure 4 by multiplying the volume, in milliliters, for V_s by 0.24.

Preparation of injection solutions

The dosage required for a slug injection is determined from equation 6 or directly from figure 4. Care should be taken to mix the contents of the container thoroughly before withdrawal. For very small quantities, graduated "To Contain" pipets should be used. Special dye pipets of tough borasilicate glass, with white graduations, are available for use in the field. They range typically from 1 to 25 mL total capacity with accuracies of ± 2 percent. Rhodamine WT 20-percent dye is rather viscous and tends to cling to the sides of graduated cylinders and other laboratory glassware; mixing with a larger quantity of water before injection will help to obtain an accurate injection of the amount measured.

Just before withdrawing the concentrated dye, 10 L or so of river water should be measured accurately into a container (most buckets will readily contain 10 to 12 L). The exact

amount of dye withdrawn (it will probably be slightly different from the amount computed) should be measured and delivered to the larger container. When using a "To Contain" pipet, the contents should be blown out into the container, care being taken not to allow droplets of dye on the outside of the pipet to spill into the container. The exact volume of rhodamine WT 20-percent dye and water mixture to be injected into the stream must be recorded.

A sample of the mixed tracer solution must be retained in the event standards are to be prepared from it; 50 mL is a desirable sample amount. The removal of the sample from the total amount injected must be noted.

Field equipment

The equipment required for a slug-dye injection is minimal; a container of stock solution, laboratory glassware for the measurement of volumes, a range of graduated pipets, a container to dilute the mixture in preparation for injection, sample bottles, sample bottle holders, a watch, and note sheets.

Glass sample bottles are recommended in preference to plastic bottles, because the dye may have a slight affinity for plastics. A convenient bottle is an 8-dr (approximately 32 mL or 1 oz) polyseal-cap glass bottle. This bottle has sufficient volume for six to eight analyses on a fluorometer and is easy to clean and handle.

Equipment required for collecting the sample will vary with the method of collection, that is, from a bridge or a boat, or by wading. The sampler shown in figure 5 holds the 1-oz bottle mentioned earlier and may be easily lowered from bridges or cableways or tossed out into the flow if necessary (fig. 6).

Generally, a fluorometer is neither needed nor desirable in the field for most dilution-type discharge measurements.

Fluorometer calibration

It is recommended that a calibration be prepared for each lot of dye and fluorometer in use, and that the results be kept in the laboratory with the fluorometer. A copy of the calibration should be provided to those doing dye-discharge measurements as an aid in selecting

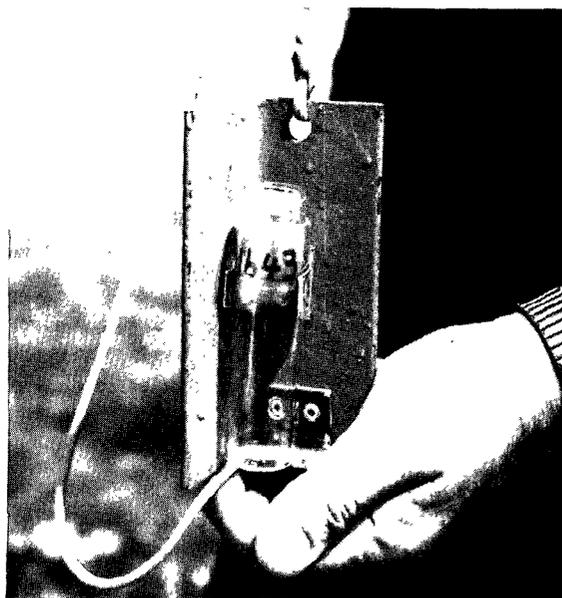


Figure 5.—Dye sampler constructed from a section of angle iron and a broom clamp.

the most suitable concentrations. For example, the graph of the fluorometer calibration in figure 7 shows that if a peak concentration of $20 \mu\text{g/L}$ were chosen, it would probably fall conveniently on the high end of scale III; thus, all smaller concentrations to be expected could also be measured using just this one fluorometer scale.

A new calibration using the serial-dilution process should be performed for any new dye lot. This procedure is explained fully in the manual on "Fluorometric Procedures for Dye Tracing" by Wilson and others (1984) and is reiterated here because the methods involved are important in understanding dilution-measuring techniques.

Serial dilution is a procedure for reducing a concentrated dye solution step by step to concentrations low enough to measure on a fluorometer. Precise measurements of dye and water in each step of the serial-dilution procedure are necessary to prepare a set of standards for accurate calibration of a fluorometer. Distilled water is usually recommended for the preparation of standards. The serial dilution procedure is based on the equation



Figure 6.—Sampling a turbulent stream by tossing a dye sampler into the flow.

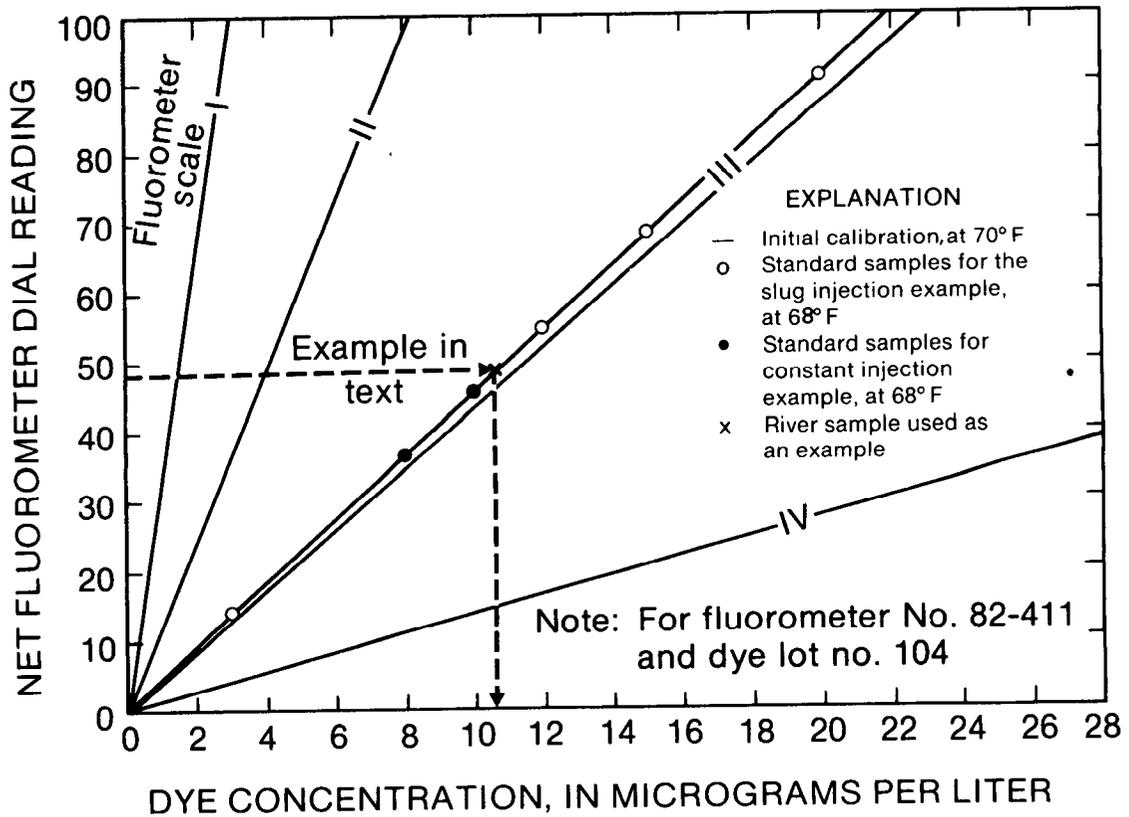


Figure 7.—Typical calibration curves for a fluorometer.

$$C_n = C_i \left[\frac{W_d}{V_w + V_d} \right] = C_i S_G \left[\frac{V_d}{V_w + V_d} \right] \quad (7)$$

where C_i is the initial concentration;
 C_n is the new concentration;
 S_G is the specific gravity of the initial dye solution, 1.19 for rhodamine WT 20-percent solution;
 V_w is the volume of the added water;
 V_d is the volume of dye solution added;
 and
 W_d is the weight of dye added.

If rhodamine WT 20 percent is used, it is necessary to perform three dilutions to obtain concentrations on the order of 100 $\mu\text{g/L}$ (see table 2A). In each step, the preceding C_n becomes the new initial concentration, C_i . It is recommended that this third solution (100 $\mu\text{g/L}$) be retained in quantity and that specific standards of smaller concentration be prepared (a fourth dilution) for the original calibration of a given dye lot as well as for the preparation of selected standards in the future when the same dye lot is used. This 100- $\mu\text{g/L}$ solution, referred to henceforth as a "working solution," should be kept sealed and stored out of direct light. The practice of retaining a working solution of 100 $\mu\text{g/L}$ for each dye lot eliminates the need to

perform a complete serial dilution each time the fluorometer is calibrated. Nevertheless, the calibration should be confirmed each time a measurement is made, as it may have changed or may be for a different sample temperature.

An infinite number of combinations of water and dye may be used to prepare standards. Table 2A provides convenient combinations for three dilutions to produce a working solution of 100 $\mu\text{g/L}$. Table 2B provides convenient combinations (columns 2 and 3) for the fourth dilution to obtain the final standard concentrations (column 4).

The following steps should be taken in analyzing standards for calibration of the fluorometer:

1. Allow all standards, including the distilled-water background samples to reach a common temperature; measurement of sample temperatures will then be unnecessary (Dunn and Vaupel, 1965).
2. Allow the fluorometer to warm up adequately, about 2 hours for the Turner Model 111 and 10 minutes for the Turner Designs Model 10.
3. Analyze all standards in the fluorometer in an identical manner: same cuvette and same temperature.

Table 2A.—Convenient three-step serial dilutions for preparation of a working solution.

(Data in milliliters except as otherwise indicated)

Dye used in test	Line	Dilutions						Working solution, in $\mu\text{g/L}$ (7)
		First		Second		Third		
		V_d (1)	V_w (2)	V_d (3)	V_w (4)	V_d (5)	V_w (6)	
Rhodamine WT (20 percent, $S_G = 1.19$)	a	50	3792	20	3500	20	3500	100
	b	25	2585	20	3000	20	3000	
	c	20	2068	20	3000	20	3000	
	d	20	1158	10	2000	10	2000	
		W_d in gms						
Pontacyl pink and acid yellow 7 (powder)	e	10	3218	20	3500	20	3500	100
	f	5	2188	20	3000	20	3000	
	g	5	1233	10	2000	10	2000	

Table 2B.—Convenient fourth-step dilutions for preparation of dye standards using a 100- $\mu\text{g/L}$ working solution.

Final standard number (1)	V_d (mL) (2)	V_w (mL) (3)	Final standard concentration C_f , ($\mu\text{g/L}$) (4)	Total dilution factor $D_T \times 10^{-8}$ (5)	Fluorometer scale			
					I (6)	II (7)	III (8)	IV (9)
1	300	100	75	31.51				
2 a	200	200	50	21.01				
b	250	250						
3 a	100	150	40	16.81				
b	200	300						
4	100	233	30	12.62				
5 a	50	150	25	10.50				
b	100	300						
*6 a	50	200	20	8.40				
b	100	400						
c	125	500						
*7 a	50	283	15	6.31				
b	100	566						
*8 a	20	147	12	5.03				
b	25	183		5.05				
c	50	366		5.05				
**9 a	20	180	10	4.20				
b	25	225						
c	50	450						
d	100	900						
**10 a	20	230	8	3.36				
b	25	288						
c	50	575						
11 a	20	313	6	2.52				
b	25	392						
c	50	784						
12 a	20	380	5	2.10				
b	25	475						
c	50	950						
13 a	20	480	4	1.68				
b	25	600						
c	50	1200						
*14 a	20	647	3	1.26				
b	25	808						
c	50	1617						
15 a	10	490	2	0.84				
b	20	980						
c	25	1225						
d	50	2450						
16 a	10	990	1	0.42				
b	20	1980						
c	25	2475						
17 a	10	1240	0.8	0.336				
b	20	2480						
c	25	3100						
18 a	10	1657	0.6	0.252				
b	20	3313						
19 a	10	1990	0.5	0.210				

* Used in example of slug-injection test.

**Used in example of constant-injection test.

4. Plot the fluorometer calibration on rectangular graph paper, and reconcile any data points that do not conform to the curve by reanalysis of fresh samples on the fluorometer and, if necessary, by preparation and analysis of new samples.

Performance

A schematic diagram of the step-by-step performance of a slug-injection type dye-dilution measurement is shown in figure 8. The steps in performing such a test use the example shown.

Selection of measurement reach (1)

The stream discharge and the other stream characteristics listed in figure 8, are estimated; using equation 4, compute an optimum stream reach length of about 700 ft. It is desirable to avoid any gain or loss in flow relative to that at the gage, so injection is to be made upstream with sampling close to the gage.

Background samples (2)

Samples of the stream water—at least one, preferably two—are needed before dye injection in order to obtain a reading of the background. The samples should be obtained just before the tracer is injected into the stream, preferably before the dye is handled at all (see sample number 202 on the note form in fig. 8).

Dye injection (3)

For the channel and flow conditions shown, it was decided to try for a peak concentration of about 20 $\mu\text{g/L}$, using rhodamine WT 20 percent. From figure 4, a volume of 0.5 mL is obtained for a peak of 1 $\mu\text{g/L}$; thus, for a peak of 20 $\mu\text{g/L}$, 10 mL of rhodamine WT dye is required.

Next, 10 L of river water is carefully measured into a clean bucket to which the 10 mL of dye is added and mixed. A 50-mL sample of this mixture is retained for future use. The entire contents (9,960 mL) are dumped as a slug into what is judged to be the centroid of the flow.

The time is noted, and the person at the sampling site is notified. A good practice is to start a stopwatch at the instant of injection so that all samples are recorded with respect to lapsed time from injection (see the notes and data plots of fig. 8).

Sampling the dye cloud (4)

The passage of the entire tracer cloud must be monitored for the slug-injection method of discharge measurement. Sampling should begin before the tracer arrives at the section and continue until it has entirely passed. Large numbers of samples are normally required to make certain the entire cloud is measured. Samples should be taken from at least three points in the section in what is estimated to be equal increments of flow. For example, if three points in the section are to be sampled, the samples should be taken at the 1/6, 3/6, and 5/6 points of cumulative flow across the section. Samples 203 through 241 are those taken to define the dye cloud at points a, b, and c across the channel.

Because the samples are not usually analyzed in the field, the length of time for complete passage is an estimate. As a rough approximation, sampling past the peak should continue for two to four times as long as it takes for the peak to arrive. In some instances, the arrival of the peak can be ascertained visually. Hence, if the peak was estimated to have arrived 5 minutes after injection, sampling should continue for a minimum of an additional 10 minutes and preferably 20 to 25 minutes. On this small stream, samples must be taken at very short time intervals; therefore two people are needed, one sampling and one noting the exact time and recording the data. Sampling may be less frequent as the dye cloud recedes. All samples are sealed and stored out of direct sunlight. Notes describing the measurement accompany the samples to the laboratory.

Fluorometric analysis (5)

The single scale that will accommodate the maximum concentrations expected in the field samples should be set on the fluorometer. The quantity of dye injected in step (3) was predicated on obtaining a peak of about 20 $\mu\text{g/L}$.

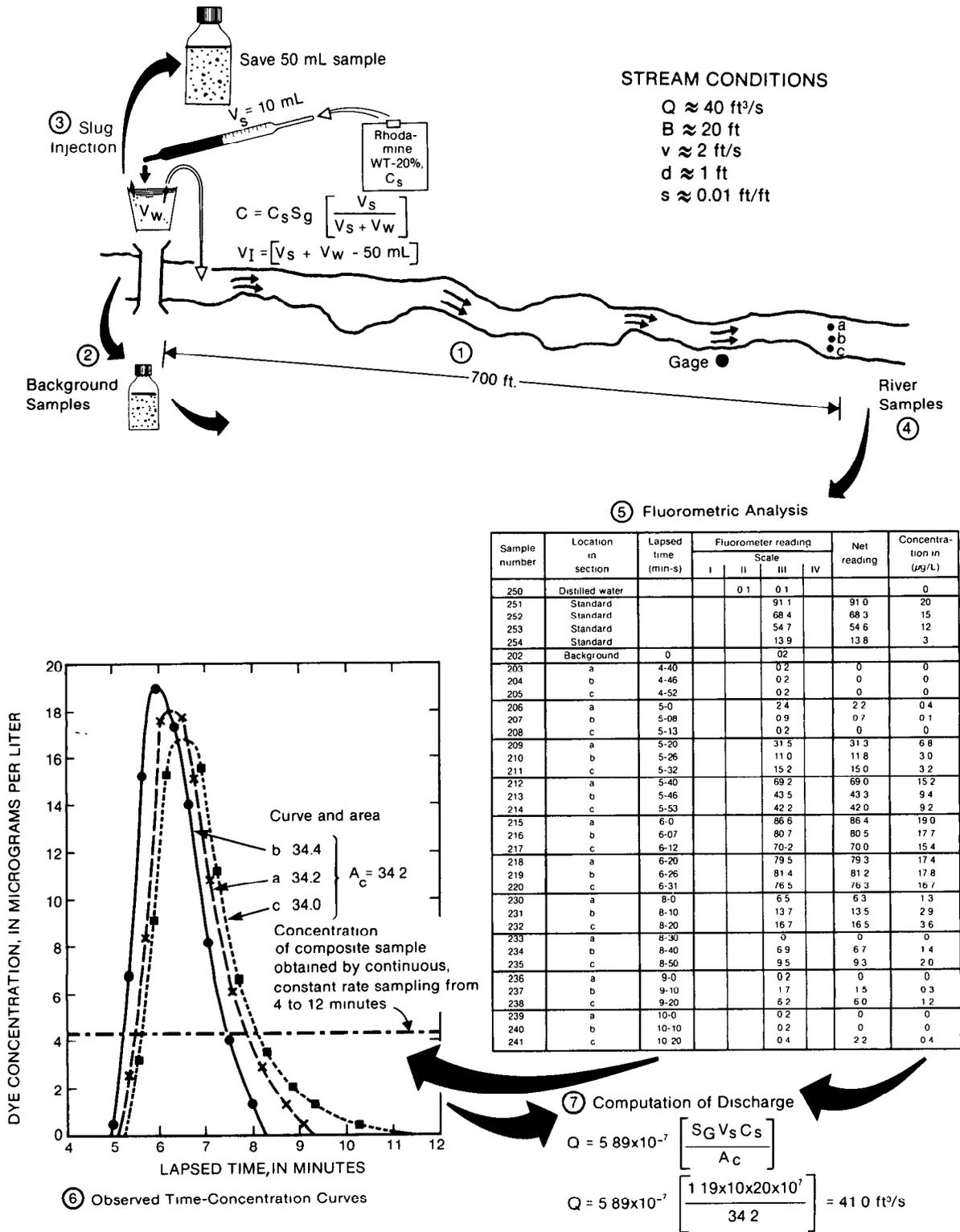


Figure 8.—Schematic diagram depicting the performance of slug-injection type dye-dilution discharge measurement.

Examination of the fluorometer calibration in figure 7 shows that it should be possible to measure that peak and all lesser sample concentrations on scale III. The use of more than one fluorometer scale should be avoided if possible.

Scale III must be recalibrated up to about 20 $\mu\text{g/L}$. Examining table 2B, four standards—20, 15, 12, and 3 $\mu\text{g/L}$ —are prepared from the 100- $\mu\text{g/L}$ working solution retained for this purpose. Those standards and all river and background samples are brought to a single temperature of 68°F. (Any temperature can be used, provided that all samples are analyzed at the same temperature.) The calibration for scale III is plotted as shown in figure 7. Note that net readings are used, the background for distilled water being subtracted first.

All river samples including the background sample, 202, are analyzed on the fluorometer, and, using the above calibration, absolute concentrations are determined for each; the stream-background reading is subtracted from each stream sample before entering the calibration curve.

Data analysis and computation of discharge (6 and 7)

The time-concentration curves for the three sample points, a, b, and c, are plotted and their areas determined. Comparison of the individual curve areas indicates that adequate mixing has been achieved. An average curve area, A_c , of 34.2 $\mu\text{g/L}$ times minutes is obtained. Using equation 3, with V_s substituted for V_1 and C_s substituted for C , the discharge may be computed as

$$Q = 5.89 \times 10^{-7} \left[\frac{1.19 \times 10.0 \times 20 \times 10^7}{34.2} \right] = 41.0 \text{ ft}^3/\text{s}^7 .$$

Alternative method of analysis and computation

During the preparation of the solution for injection, 10 mL of dye was mixed with 10 L of river water, and a 50-mL sample of the mixture was retained (step 3, fig. 8). The concentration used in the earlier computation was for the manufacturer's stock solutions: $20 \times 10^{-7} \mu\text{g/L}$

for rhodamine WT. Using equation 7, it can be seen that the 10 L had a concentration of

$$C = 1.19 \times 20 \times 10^7 \left[\frac{10.0}{10,000 + 10.0} \right] = 23.8 \times 10^4 \mu\text{g/L} .$$

Hence, a major dilution was made at this point. Thus, the 50 mL sample may be used in preparing standards for this particular measurement instead of starting with the stock solution.

In preparing standards from C (above), note that, according to equation 7, 20 mL of dye solution added to 1,980 mL of water is a dilution of 10^{-2} . If repeated, a total dilution of 10^{-4} is obtained, because the final dilution in any serial dilution is the product of the individual dilutions. For any dilution after the first, the specific gravity term in equation 5 may be ignored. Thus, if C (above) was diluted twice, as described, for a total dilution of 10^{-4} , the resulting concentration would be $23.8 \times 10^4 \times 10^{-4} = 23.8 \mu\text{g/L}$. While this would be off scale III in figure 7, other standards may be produced by judicious selection of dye and water volumes. The analysis in figure 8 would then be based on the fluorometer calibration thus obtained, and computations would be based on the new volume and concentration injected such that the discharge (using eq. 3) would be

$$Q = 5.89 \times 10^{-7} \left[\frac{(10,000 + 10 - 50) \times 23.8 \times 10^4}{34.2} \right] = 40.8 \text{ ft}^3/\text{s} .$$

This alternative method of analysis is more accurate than the first, because any errors that occur when measuring the 10 mL of dye for use in the solution to be injected are canceled. Similarly, in preparing standards, only two dilutions are necessary, instead of four as in the previous example. Furthermore, if the exact concentration of the solution injected is unknown, a measurement is still possible if a sample of the solution that was injected is retained and used in the laboratory to develop a calibration in the manner previously described. Such a trial-and-error calibration would be more work in the laboratory but would be accurate if correctly done. That is true because the

concentrations that would be plotted for the time-concentration curves of figure 8 are based on a calibration which is, in turn, based on the injected concentration, C ; hence, the exact concentration of C is not needed. Knowledge of the absolute value of C does simplify the preparation of standards. The disadvantage of the alternative method is that more standards must be prepared, as well as new ones for each measurement. Obviously, it is important that the field person always retain a sample of the actual dye solution injected.

Alternative method of measurement

As the data in figure 8 show, 39 samples were required to define the three time-concentration curves. An alternative method of sampling greatly reduces the sampling requirements but may obscure the quality of the data and the accuracy of the measurement. This alternative method consists of withdrawing a sample continuously and at a constant rate from the stream during the passage of the dye cloud. The time period of sampling, ΔT_c , should be greater than the actual time, T_D , the dye cloud is present. Each sample obtained for the one or more points of sampling is composited, and one sample is analyzed for concentration. This concentration, less the background, is the average concentration, C_a , for the entire dye cloud for the time, ΔT_c . Therefore, $A_c = \Delta T_c C_a$, and equation 3 may be written as

$$Q = 5.89 \times 10^{-7} \frac{S_G V_I C}{\Delta T_c C_a} \quad (8)$$

Assume for the example used above that a sample was pumped at a constant rate from the center of the channel into a clean container for an 8-minute period. If this composite sample yielded a concentration of $4.30 \mu\text{g/L}$ (see fig. 8), the discharge may be computed as

$$Q = 5.89 \times 10^{-7} \left[\frac{(10,000 + 10 - 50) \times 23.8 \times 10^4}{8 \times 4.30} \right] \\ = 40.6 \text{ ft}^3/\text{s}$$

For this example, the assumption was made that mixing was complete. An alternative to pumping a sample would be to collect manually, individual samples of a given volume at uniform time intervals to form a composite sample. Note that for the above example, any time interval greater than 10 seconds might not adequately sample the peak concentrations and thus might not yield a representative sample. From a practical standpoint, it would be difficult to sample manually at such frequencies at more than one point in the section. Where this method is used, sampling just at one point in the main flow at a greater length (say 1,000 ft in the above example) might be advisable. Not only would mixing be improved (and therefore samples at just one point more likely to yield accurate results), but sampling at uniform 15-second intervals would probably be sufficient.

Note that in the example illustrated in figure 8, adequate and complete measurement of all three of the time-concentration curves requires very frequent sampling and accurate recording of time of sampling. In fact, the task would be virtually impossible for one man to perform; one man is needed to sample and one to record the exact times of sampling. In this case, a reach length of 1,000 ft or more might have been advisable to elongate the dye cloud for greater ease and accuracy of sampling. On larger streams with correspondingly greater mixing lengths, this would not be as serious a problem.

Constant-Rate Injection Method

Overview

The constant-rate injection method generally is preferred to the slug-injection method in spite of the additional requirement for a device that will release the tracer at small, constant rates. Various devices that will achieve the required small, constant rates of injection are available commercially or can be fabricated. They are described in detail later in this

manual. An advantage of the constant-rate injection method is that significantly fewer samples are needed, compared with the slug-injection method.

Equation 2 is in a more usable form as

$$Q = 5.89 \times 10^{-7} q \frac{C}{\bar{c}} \quad (9)$$

where Q is the discharge of the stream, in cubic feet per second;
 q is the rate of injection of the tracer, in milliliters per minute;
 C is the concentration of the dye solution injected into the stream, in micrograms per liter; and
 \bar{c} is the equilibrium, or plateau concentration, in micrograms per liter, averaged or weighted across the sampling section.

This equation not only is used to compute the discharge measurement, but also, by solving for q for a given c , may be used to estimate dye quantities for preparing injection solutions. Mixing distances and reach selection are determined in the same manner as for the slug-injection method.

Preparation

Selection of dye concentrations and injection rates

The curves in figure 9 were computed from equation 9 for a plateau concentration, \bar{c} , of 10 $\mu\text{g/L}$. A dye concentration of 10 $\mu\text{g/L}$ in the stream is well above background levels commonly experienced. In most cases, a good dye-dilution discharge measurement can be made

NOTE: Curves are based on equation $Q = 5.89 \times 10^{-7} q \frac{C}{\bar{c}}$ for $\bar{c} = 10 \mu\text{g/L}$;
 greater discharges may be measured by allowing \bar{c} to be less than 10 $\mu\text{g/L}$.

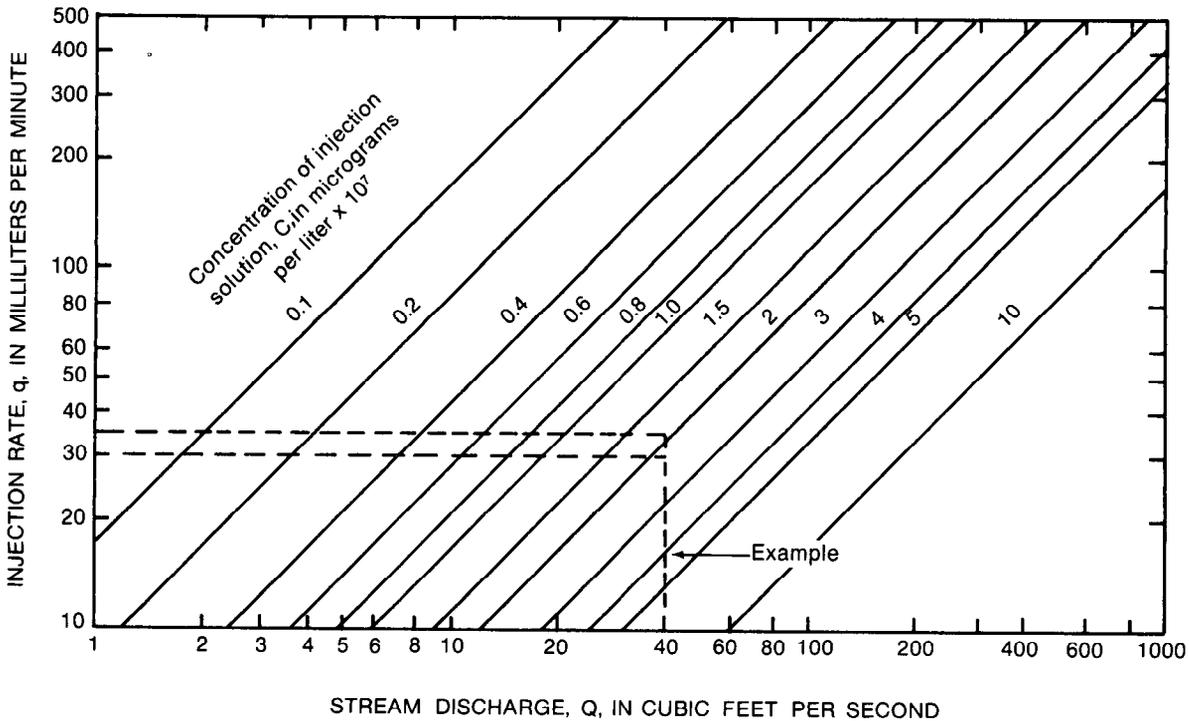


Figure 9.—Graph used for estimating rates of dye injections for different stream discharges and concentrations of injection solutions.

Table 3.—Convenient volumes of water and dye to mix for the preparation of bulk solutions of selected dye concentrations.

	Desired concentration C in $\mu\text{g/L} \times 10^7$ (1)	Volumes to mix together		Dilution factor $D_C \times 10^{-2}$ (4)	Volume of mixture in gallons (5)
		V_w Water, in milliliters (2)	V_d 20% Rhodamine WT dye, milliliters (3)		
1	10	10,700	7,750	42.0	4.87
2	10	11,000	8,000	42.1	5.02*
3	5	14,100	3,750	21.0	4.72
4	5	15,000	4,000	21.1	5.02*
5	4	14,850	3,000	16.8	4.72
6	3	15,600	2,250	12.6	4.72
7	2	16,350	1,500	8.40	4.72**
8	1.5	16,350	1,100	6.30	4.61
9	1.5	17,850	1,200	6.30	5.03*
10	1.0	17,100	750	4.20	4.72
11	0.8	17,250	600	3.36	4.72
12	0.6	17,400	450	2.52	4.72
13	0.4	17,550	300	1.68	4.72
14	0.2	17,700	150	0.80	4.72
15	0.1	17,775	75	0.42	4.72

* In many instances, commercial 5-gallon containers are slightly greater in actual volume; the smaller volumes allow for easier mixing.

**Used in example.

with concentrations as low as 2 $\mu\text{g/L}$; however, higher concentrations are preferred, to lessen the significance of background levels and thus to improve accuracy. Figure 9 is used only to estimate the injection rate needed; the allowable latitude in \bar{c} virtually guarantees a successful measurement, no matter how poor the estimate may be. Furthermore, if 2 $\mu\text{g/L}$ is viewed as a lower limit for \bar{c} , discharges five times those in figure 9 may be measured, using the concentrations and injection rates shown.

Preparation of injection solutions

Bulk solutions

Frequently, the streams to be measured are readily accessible by vehicle, and several streams are to be measured by dye-dilution methods on a single field trip. An example might be a series of turbulent mountain

streams where current-meter measurements are difficult or impossible to make. In such a case, considerable field and laboratory work may be saved by preparing one or two solutions of dye in advance in bulk. The solutions should be tailored to the expected range of flows and the capabilities of the injection apparatus.

Table 3 shows convenient combinations of volumes of rhodamine WT 20-percent dye and water to yield about 5 gal of bulk solution with the concentrations shown in figure 9. Five gallons is normally sufficient for a dozen discharge measurements. The dilutions for concentrations in column 1 of table 3 were determined using equation 7. That equation permits the preparation of any smaller concentrations of dye, such as for the family of curves in figure 9, based on measured combinations of water and dye. Thus any concentration of dye desired can be obtained to suit the need, depending on the

Table 4.—Prepackaged dye quantities to produce selected concentrations of injection solutions.

	V_d , Premeasured volumes of rhodamine WT 20% dye, milliliters (1)	Number of bottles of dye (2)	$V_w + V_d$, Total volume of water and dye solution, milliliters (3)	Dilution factor $D_C \times 10^{-1}$ (4)	Resulting solution concentration, $C, \mu\text{g/L} \times 10^7$ (5)
1	50	1	2000	0.25	0.60
2	50	1	1000	0.50	1.19
3	100	2	2000	1.00	2.38
4	100	1	1000	1.00	2.38*
5	100	2	1000	2.00	4.76
6	100	4	2000	2.00	4.76
7	100	5	2000	2.50	5.95
8	100	6	2000	3.00	7.14
9	250	1	1000	2.50	5.95
10	250	2	1000	5.00	11.90

*Example

capability of the injection equipment and the stream discharge to be measured. The assumption has been made that rhodamine WT 20 percent has a specific gravity of 1.19. Tests have indicated that this may vary from 1.10 to 1.20. For a given dye lot, any such variation will not affect the accuracy of the measurement; the error is canceled, for in equations 2 and 9 the concentrations form a ratio and are relative. This will become apparent later in the discussion of the use of relative concentrations. The dye as received from the manufacturer in large containers should be well mixed before extracting dye to prepare injection solutions. Carefully roll the drums to mix any solid dye that may have been deposited on the bottom.

The advantages of bulk preparation of the dye solutions are

1. Field preparation is unnecessary.
2. Clean water may be used in solution preparation, thereby preventing impurities from causing injection stoppages.
3. The fluorometer may be calibrated in advance for the dye mixture used and only spot-checked at the time of analysis of the stream samples.

4. Many measurements can be made from one concentration of injection solution, thus greatly reducing laboratory preparation of samples and calibration of the fluorometer.

Prepackaged dye quantities

Dilution gaging is particularly well suited to measuring discharge on small cobble-strewn or rocky mountain streams. Such streams are frequently inaccessible, and carrying quantities of premixed dye solutions is onerous. An alternative is to prepackage carefully measured quantities of dye to permit the preparation of injection solutions onsite, using stream water. Table 4 provides a convenient range in premeasured dye quantities to be mixed with selected volumes of water to obtain desired concentrations of injection solutions. These solutions should be packaged in bottles of suitable size, sealed with plastic tape, and labeled. The total volumes of injection solutions have been made to be 1 or 2 L to permit use of those sizes of laboratory graduated cylinders, both for measuring the quantity of solution and as calibrated reservoirs from which injection rates can be measured.

Constant-rate injection equipment

The constant-injection dye-dilution method requires apparatus capable of injecting dye at a small, constant rate reliably and accurately. Equipment that can be purchased or fabricated that will produce constant-rate injections under field conditions includes battery-driven pumps, mariotte vessels, and various chemical-feed devices; each is discussed here.

Pumps

Figure 10 shows a small fluid-metering pump operating on a 12-volt d-c battery withdrawing a dye solution from a graduated cylinder. It is a valveless, variable, positive-displacement pump that can be set for a rate up to 48 mL per minute. Different models are available in a range of capacities and produce accuracies on the order of 1 percent. The valveless feature is desirable because it is cleaner and can handle dirt and foreign material in the dye solution. The rate settings have been found to be reproducible within a fair degree of accuracy; nevertheless, the actual injection rate should be independently measured. Figure 10

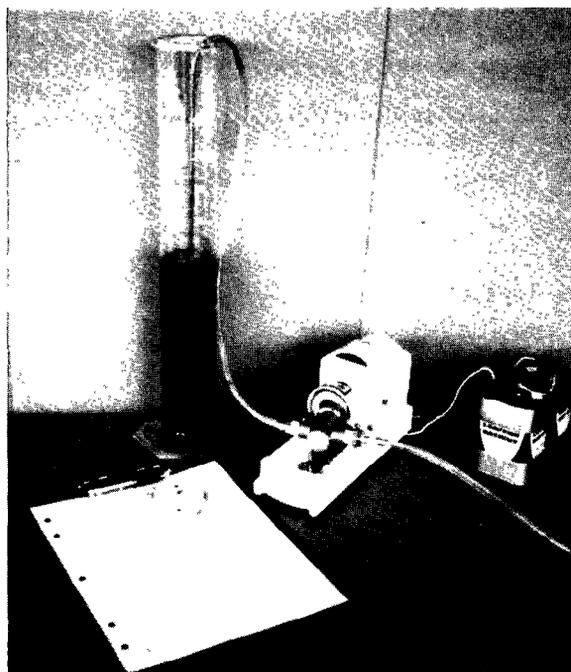


Figure 10.—Pump and graduated cylinder used for constant-rate dye injection.

shows this measurement being done volumetrically by plotting volume versus time and calculating the injection rate from the slope of the resulting line. The advantages of this method of measuring the injection rate are that

1. Any change in the injection rate during the test will be revealed by a change in the slope of the line.
2. The injection is not interrupted during the discharge measurement and may be observed throughout.
3. Sufficient data are obtained to guarantee an accurate measurement of the injection rate.
4. Separate sampling of the dye-injection rate is not necessary, thus reducing the chance of contamination of the hands; this is particularly important if only one person is performing the discharge measurement.

Thin-wall, 1/8-in-diameter brass tubing is used in the cylinder as a withdrawal line to prevent an error in the measured volumes. The pump shown in figure 10 is designed for pumping against a relatively low pressure of 5 lb/in². For pumping into pipes and under ice, and for multiple-line injections, pumps capable of injecting against greater pressures are available. Regardless of manufacturers' ratings, all pumps should be bench tested to check rates and stability of each rate.

This apparatus may be packaged into a backpack unit for use at remote sites (see fig. 11). The entire unit, shown closed in figure 12, weighs only 20 lb. It can be set up, as shown, in minutes. After each use, a water or water-and-alcohol solution should be pumped through the unit to prevent a residue of dye from drying inside and coating parts and tubes.

Peristaltic pumps have also been successfully used for the constant injection of dye solutions. They are usually less expensive than the displacement-type pump but have not proven as accurate.

Mariotte vessels

The mariotte vessel is an inexpensive, simple, and yet reliable constant-injection device. Virtually any rigid, sealable vessel can be made into a mariotte vessel (see fig. 13). The



Figure 11.—Backpack dye-injection apparatus in use.



Figure 13.—Dye injection using a 5-gal can mariotte vessel positioned over the stream by a light cable and trolley system.

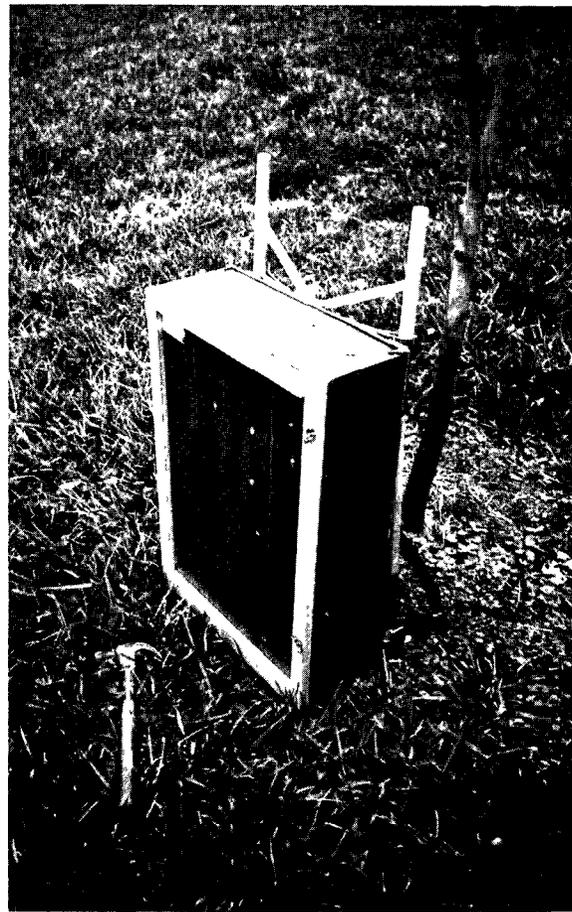
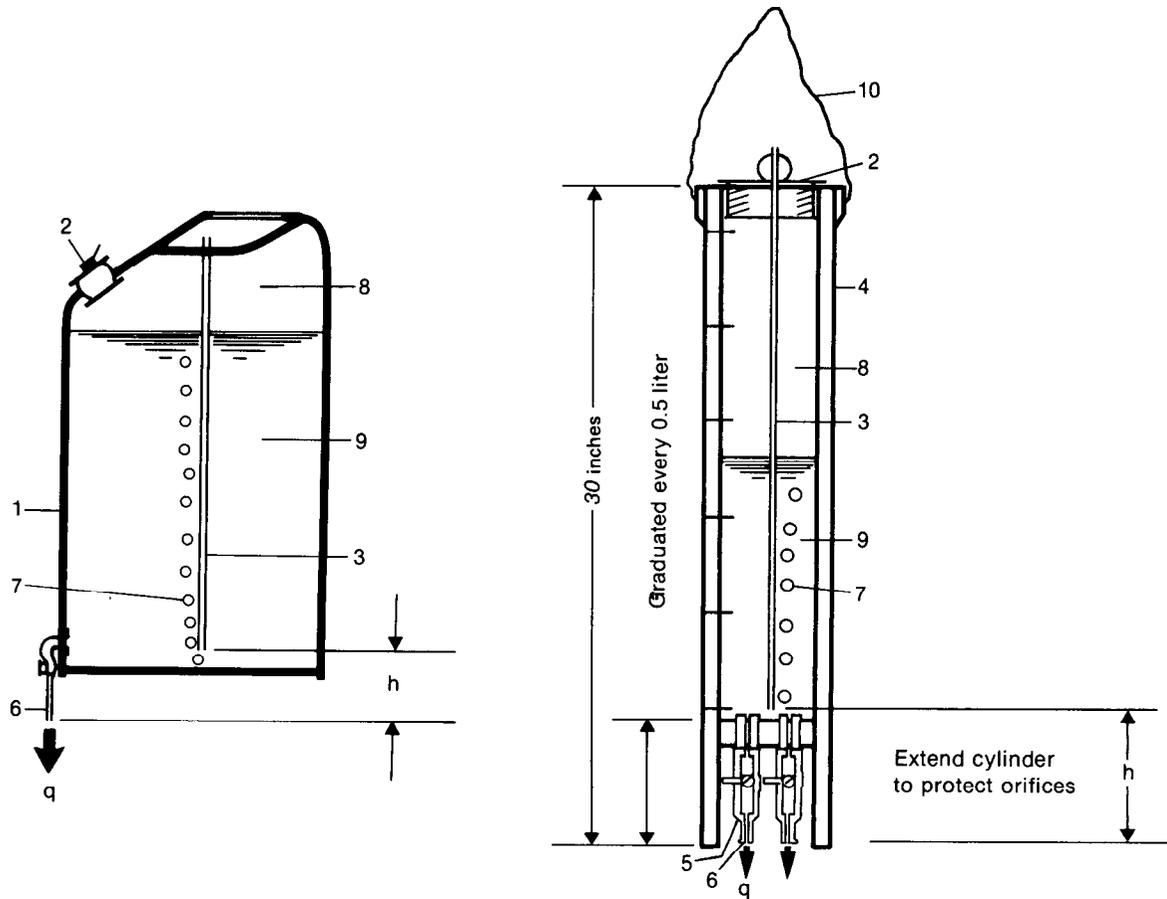


Figure 12.—Backpack dye-injection apparatus ready for transport.

device works on the principle that the air entering the tank to replace the outgoing dye solution is at all times at atmospheric pressure at the tip of the air-intake tube (see fig. 14). Thus, whatever the depth of fluid in the sealed container, constant pressure exists at the tip of the air-intake tube regardless of its depth. The dye solution discharges through a bottom tube or orifice at a rate depending on outlet orifice size and head, h , on the orifice. The head can be varied by raising or lowering the elevation of the outlet tube relative to the bottom of the air-intake tube. The mariotte vessel is a reliable constant-rate injection device if these conditions are ensured:

1. The dye solution is clean and free of dirt that might clog the orifice; it is sometimes desirable to strain the solution.



EXPLANATION

1. Water can, 5-gallon, with heavy enameled interior.
 2. Vacuum bottle top, or plumber's 4-inch pipe plug, expandable type; all openings must be sealed airtight.
 3. Air-inlet tube.
 4. Plastic cylinder, rigid, 4-inch inside diameter.
 5. Gas valve, brass.
 6. Orifice, brass; a range of sizes is used to match desired injection rate.
 7. Air emitted to replace discharging dye solution; atmospheric pressure exists at low point of air tube regardless of fluid above it.
 8. Airspace above dye solution; a partial vacuum exists, requiring that container be of rigid-wall, airtight construction.
 9. Dye solution.
 10. Sling.
- h. Head on orifice; equal to difference in elevation between tip of orifice and bottom of air tube.
- q. Injection rate; a function of the diameter, D , of the orifice, and of h . An approximate equation for the injection rate is $q(\text{mL}/\text{min}) = 50,000 D^2 \sqrt{h}$ where D and h are both in inches.

Figure 14.—Design drawing of mariotte-type, constant-rate injection apparatus.

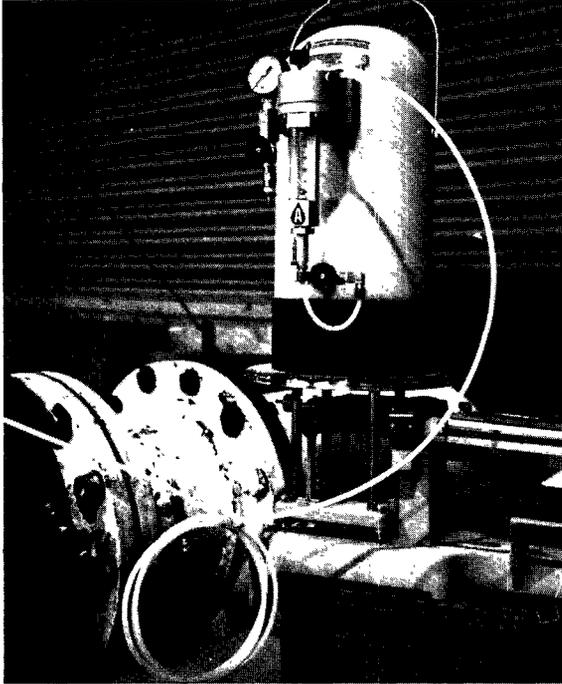


Figure 15.—Pressurized chemical-feed tank being used to inject dye into a pipeline. A laboratory balance measures the injection rate.

2. Air bubbles do not block the orifice or cause a change in rate by collecting in the discharge tubing. Air bubbles can be avoided by using warm water to prepare the dye solution and by shielding the vessel and tubing from sunlight.
3. Orifices and tubing with larger diameters are used, yielding fairly high injection rates and thus transporting dirt and bubbles with less chance of blockages. A minimum injection rate of about 60 mL/min is recommended. Figure 14 shows that, by using a uniform clear plastic cylinder to construct the mariotte vessel on the right, the cylinder can be graduated and injection rates measured by observing change in volume with time. Figure 10 shows this procedure being done with a graduated cylinder.

Chemical-feed devices

Many types of chemical-feed devices are in use to inject chemicals into water and wastewater treatment systems. Most operate on 110 volts

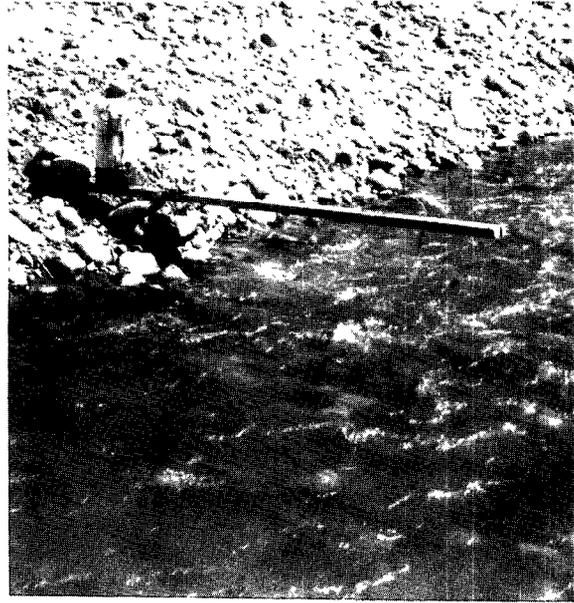


Figure 16.—Single-point, midchannel dye injection from a chemical-feed tank.

and are unsuited for field use. Figures 15 and 16 show a fiberglass chemical-feed tank powered by air pressure that was designed originally to inject chlorine into remote water supplies like those at parks. Such tanks have been used with some success for the constant-rate injection of dye solutions. The main disadvantage of the equipment has been the problem of keeping the rate controller clean. The device must be thoroughly flushed after each use and the controller unit dismantled and cleaned after several uses. The unit is powered by air pressure, and certain safety precautions must be observed. The advantages of the unit are reliable constant-rate injection and the capability of injecting under pressures up to 50 lb/in². This feature is particularly valuable when injecting into a pipeline, as illustrated in figure 15; in this figure, the injection rate is shown being measured with a laboratory balance by observing the change in weight of the dye solution with time.

Performance

A schematic diagram of the performance of a constant-rate dye-dilution measurement is

shown in figure 17. The steps in performing such a measurement use the example shown.

Selection of measurement reach (1)

The stream discharge, mean velocity, and geometry are estimated; using equation 4, an optimum reach length of 700 ft is computed. If there is any question of gain or loss of flow in the stream, or if stage is changing rapidly, an injection is made about 700 ft upstream of the gage and the samples taken close to it.

Background samples (2)

One or two stream background samples are taken before handling dyes and injection equipment.

Dye preparation, injection, and sampling (3)

For an estimated streamflow of 40 ft³/s, figure 9 shows that an injection rate of about 35 mL per minute of a 2.0×10^7 µg/L (2-percent) solution should yield a plateau concentration of about 10 µg/L. Referring to table 3, a 2-percent bulk solution could be prepared by mixing those quantities of water and dye shown in columns 2 and 3, line 7. For a single measurement, just the quantity of solution needed would be preferable. Referring to line 4, column 5, of table 4, it would be convenient to use a 2.38×10^7 µg/L solution with an injection rate of about 30 mL per minute. Thus 1,000 mL would allow injection for 33 minutes. That is usually sufficient on most small streams, but if more time were needed, a 2,000-mL cylinder could be used, adding two 100-mL bottles of dye rather than one, to obtain the necessary dye concentration.

The injection solution is prepared by first emptying a 100-mL bottle of dye into the 1-L cylinder and adding stream water until a total volume of 1,000 mL is obtained. Before topping off the graduated cylinder with river water, it is a good practice to rinse the bottle that had contained the dye with river water, emptying it several times into the cylinder. Then the cylinder should be carefully filled to the 1,000-mL mark. Precision of measurements at this time will make it unnecessary to prepare standards from a sample of this solution, although a sample should be retained in the event it is needed.

The dye mixture in the cylinder should be thoroughly mixed and allowed to stand for a few minutes.

The intake line to the pump is secured inside the graduated cylinder to reach nearly to the bottom. The plastic discharge line leading from the pump should be positioned and secured where the injection will enter what is judged to be the centroid of flow. The pump is turned on and set to a rate of about 30 mL/min. At the end of the injection—after steps (4) and (5)—a sample of injected solution should be bottled, labeled, and stored separately from the stream samples.

Measurement of injection rate (4)

As soon as the injection rate has stabilized, the change in the volume of dye-solution level in the cylinder should be timed with a stopwatch. The watch is not stopped, but the time and volume are observed simultaneously several times before and after stream samples are collected downstream. These volumes are plotted versus time and the rate determined from the slope of the line as 31.2 mL/min.

Sampling the plateau (5)

In contrast to the slug-injection method, immediate and complete sampling of the dye downstream is neither necessary nor desirable. The injection must be maintained at a constant rate long enough for plateau concentrations to be fully established laterally across the channel and with time. The best practice is to inject longer and sample later than is likely needed. Sampling should, of course, be done toward the end of the injection but before its termination.

Stream samples containing the diluted dye should be taken at a minimum of three points across the flow at the sampling section (labeled a, b, and c in fig. 17). Additional sampling points may be advisable if the channel is wide and shallow, or if mixing is thought to be less than optimum.

All samples should be sealed, labeled with the stream name, sampling location and time, and stored out of direct sunlight. Notes describing the measurement should be retained with the samples and sent with them to the laboratory.

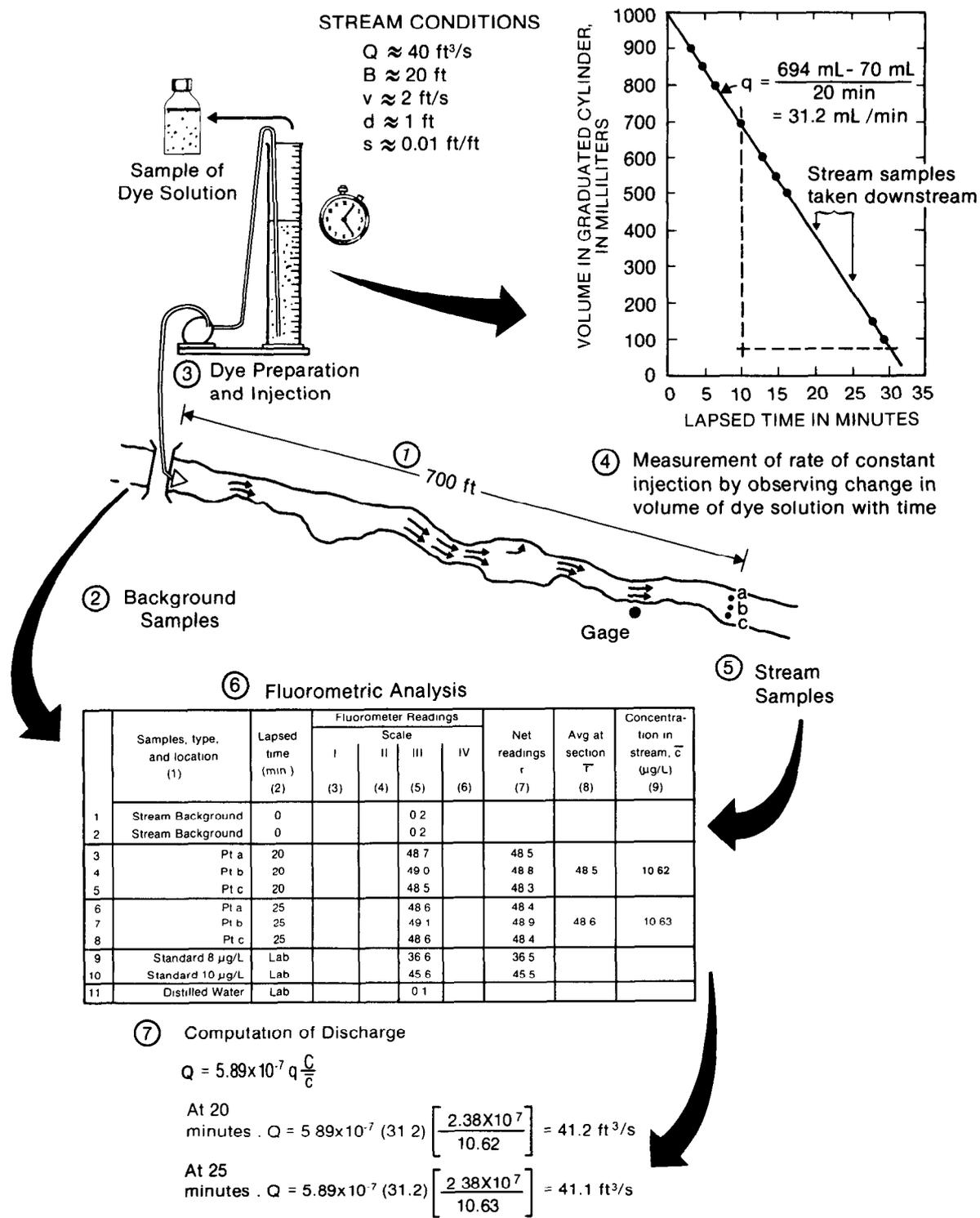


Figure 17.—Schematic diagram of the performance of a constant-rate dye-dilution discharge measurement.

Fluorometric analysis (6)

Assuming that c values at sampling points a , b , and c will be on the order of $10 \mu\text{g/L}$, standards in that concentration range are required. A working solution of $100 \mu\text{g/L}$ should be prepared from the dye lot used and stored for future use. Thus, according to lines 9c and 10c of table 2B, standards of 8 and $10 \mu\text{g/L}$ are prepared by mixing 50 mL of $100\text{-}\mu\text{g/L}$ solution in each of two containers containing 575 mL and 450 mL of distilled water, respectively. The 8- and $10\text{-}\mu\text{g/L}$ standards and a background sample of the distilled water used in their preparation are placed in sample bottles similar to those used for collecting field samples. All stream samples, stream background, standards, and distilled-water background samples are brought to a common temperature— 68°F in the example. All samples are analyzed on one fluorometer scale.

The net dial readings for the 8- and $10\text{-}\mu\text{g/L}$ standards are plotted on the existing fluorometer calibration after subtracting the dial readings for the distilled water sample. If the readings do not plot exactly on the established calibration curve, a new curve is drawn through the two points for that scale parallel to the original calibration (see fig. 7). The primary reason for the shift from the original calibration curve is the difference in sample temperature from that used in the original calibration. The net dial readings for the stream samples are averaged, and, using the new calibration curve, the net concentrations of the stream samples for each set are determined, as shown in column 9 of figure 17. In this case, an average plateau concentration, \bar{c} , of 10.62 and $10.63 \mu\text{g/L}$ was obtained for the sets taken at 20 and 25 minutes.

Computation of discharge (7)

Based on the data presented and using equation 9, the discharge may be computed as

$$Q = 5.89 \times 10^{-7} (31.2) \left[\frac{2.38 \times 10^7}{10.62} \right] = 41.2 \text{ ft}^3/\text{s}$$

for the samples taken 20 minutes after the start of injection and 41.1 cubic feet per second for samples taken after 25 minutes.

Discussion

The example of a constant-rate dye-dilution measurement has been kept rather simple. Frequently, those attempting to apply the method for the first time will overly complicate the procedures. The use of fluorometers in the field, either to analyze individual samples or in a pump-through mode such as is done in time-of-travel studies, is discouraged. Aside from the added labor involved, most fluorometers are not sufficiently accurate under field conditions. Suspended sediment and varying sample temperatures are sources of error not acceptable for a discharge measurement.

Estimation of the mixing length and selection of sampling time are critical. In practice, three samples taken laterally are a minimum; five are advisable, especially if the stream is wide and the flow is shallow. Similarly, in the example cited, sampling at 1,000 ft would be advisable if there were any question regarding the adequacy of mixing length. For a 1,000-ft distance, samples might need to be taken at 30 minutes or later, and the injection continued as necessary.

It is not possible to inject too long or to sample too late, assuming that in the latter case the rate is constant. Figure 3 shows that if samples are taken too soon, the concentrations along one or both banks will be measured prematurely, yielding low values compared to the correct plateau concentrations. Thus the computed discharge would be high. This might suggest loss of dye when in reality sampling was done too soon. Similarly, this much variation in concentration laterally might suggest poor mixing, whereas waiting longer to sample would have revealed good mixing. Thus it is emphasized that injection should be long enough and sampling late enough to ensure that what is being measured is a fully developed concentration plateau across the stream and with time.

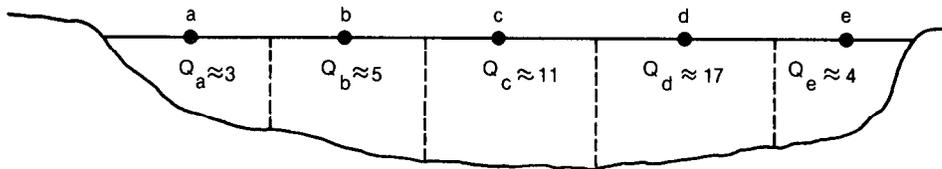
In the example, the concentration of the injection solution was chosen for a discharge estimated at $40 \text{ ft}^3/\text{s}$. Had the discharge actually been $60 \text{ ft}^3/\text{s}$, the stream samples would have had a concentration of about $7 \mu\text{g/L}$. Such concentrations are more than adequate for an accurate measurement. All samples are best ana-

lyzed on the same scale of the fluorometer; an additional standard of about 7 µg/L would have been desirable. For convenience, a 6-µg/L standard could be prepared, working from line 11c of table 2B.

Measurements with less than optimum mixing

The nearly identical concentrations observed at 700 ft in the previous example indicate that

mixing is adequate and the mixing distance probably optimum. If a reach length of 400 ft had been chosen, mixing probably would not have been as complete. As a further example, it is assumed that the flow at the 400-ft sampling section is distributed as shown in figure 18 and that dye concentrations at a through e are fully stabilized with time as shown. Equation 2 is predicated on c as a measure of the dilution throughout the flow section, which is not the case at this short distance. If data reveal that



Cross-section and approximate discharge distribution at 400 feet

Location of samples taken at section 400 feet downstream (1)	Fluorometer reading for stream samples (2)	Stream background reading (3)	Net reading (4)	Estimated portion of total discharge (5)	Product of columns 4 and 5 (6)
Pt a	32.2	0.2	32.0	3	96
Pt b	42.2	0.2	42.0	5	210
Pt c	58.6	0.2	58.4	11	642.4
Pt d	40.2	0.2	40.0	17	680
Pt e	37.4	0.2	37.2	4	148.8

Note: all readings on same fluorometer scale

Σ = 40

Σ = 1777.2

Arithmetic average = 41.9

From figure 7, \bar{c} = 9.20 µg/L

Weighted average scale III reading for \bar{c} = $\frac{1777.2}{40}$ = 44.4

From figure 7, \bar{c} = 9.80 µg/L

$$Q = 5.89 \times 10^{-7} q \frac{C}{\bar{c}}$$

Using

Arithmetic

Average: $Q = 5.89 \times 10^{-7} (31.2) \left[\frac{2.38 \times 10^7}{9.20} \right] = 47.5 \text{ ft}^3/\text{s}; 16 \text{ percent high}^*$

Using

Weighted

Data: $Q = 5.89 \times 10^{-7} (31.2) \left[\frac{2.38 \times 10^7}{9.80} \right] = 44.6 \text{ ft}^3/\text{s}, 8 \text{ percent high}^*$

*Assuming 41.1 ft³/s to be the correct discharge.

Figure 18.—Example of flow-weighting concentration data where mixing is poor.

mixing is not as good as desired (appendix A contains a computation of the percentage of mixing for this example), a more correct discharge can be computed if the concentrations at each lateral sampling point are discharge weighted. Thus it is a good practice to estimate and assign a proportion of the total flow to each sampling point and weight the data during the computation as shown in figure 18. Although the weighting procedure is recommended, it should not be used as a substitute for trying to select an adequate stream reach to produce optimum mixing.

Appendix B contains a form for the recording of field data, fluorometric analysis of samples, and computation of discharge using the constant-injection method incorporating the above weighting procedure. The data used in the figure 18 example are shown in a completed form to help the user.

As indicated, dilution-type measurements of discharge can be made where mixing is less than optimum if samples are flow weighted across the stream. This can also be accomplished by using a depth-integrating sediment sampler to obtain a number of equally spaced, vertically integrated samples across a stream, as is standard practice in sediment sampling (Guy and Norman, 1970). A single composite sample is analyzed to obtain \bar{c} . The number of verticals required will depend on the degree of mixing that has taken place at the sampling section and on the accuracy requirements for the discharge measurement. This method can be approximated by moving the integrating sediment sampler laterally at a constant speed at approximately middepth. These methods generally should not be used where mixing is less than 75 percent, as there is some subjectivity in getting depth-integrated samples.

The Use of Relative Concentrations

It is apparent from equation 9 that concentrations could be measured in any similar units, since C/c is a ratio—for example, fluorometer dial readings. Furthermore, if both river samples and standard samples are analyzed in an identical manner, certain potential

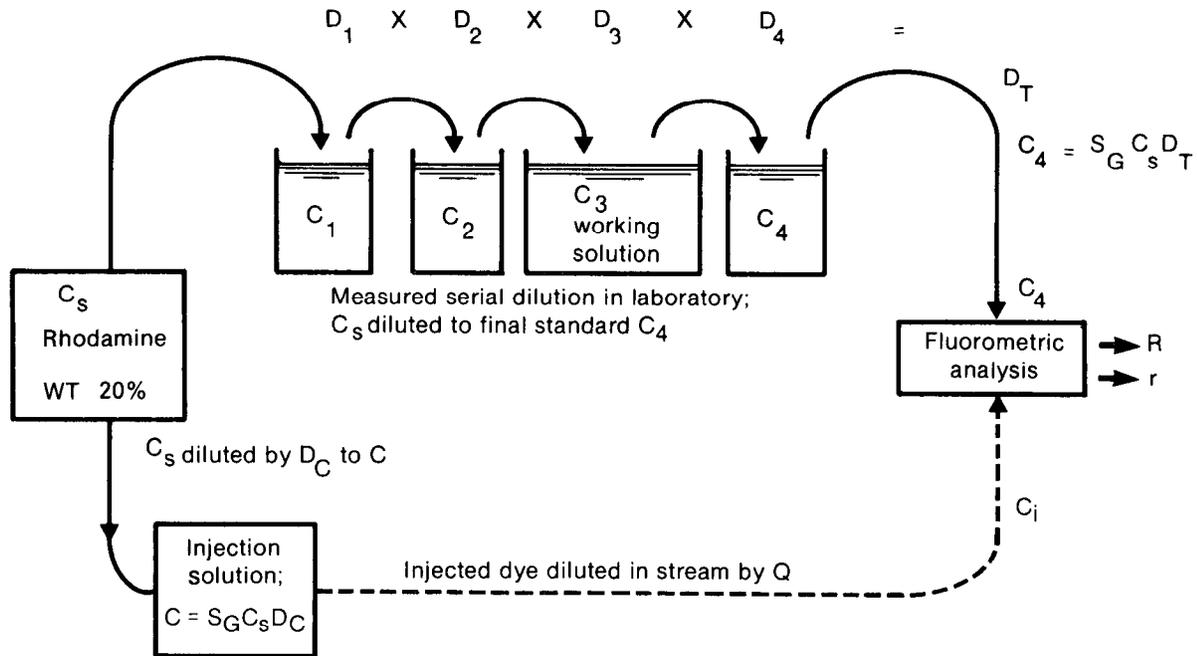
errors may be canceled or lessened. This is one of the reasons for placing river samples and standards samples in identical bottles and allowing all samples to come to a common temperature. For the same reason, it is recommended that all samples be analyzed on the same fluorometer scale. As has been noted, the 8- and 10- $\mu\text{g/L}$ standards were chosen to coincide closely with the concentrations expected for the stream samples. This was done to ensure that all samples would be analyzed on the same fluorometer scale, thereby lessening any errors that might occur by shifting from one fluorometer scale to another, as well as for simplicity. Similarly, the fluorometer calibration should be prepared from the same dye lot as used in the actual field test.

Two approaches to the use of relative concentration are presented; both have advantages and disadvantages as to simplicity and accuracy. Both use net fluorometer dial readings to evaluate the ratio C/\bar{c} in the constant-rate dye-dilution discharge relationship, equation 9. Both require the computation and use of dilution factors. The first case requires the total dilution factor, D_T , necessary in reducing the manufacturer's stock solutions, C_s , down to a final standard solution and in the second case the single dilution factor, D_C , necessary in reducing C_s down to C , the concentration of the dye solution that was injected. The second case requires that the determination start with the injected solution rather than the manufacturer's solution. The schematic diagrams in figures 19 and 20 aid in understanding the concept and use of relative concentrations for computing the discharge where the constant-injection method is used. The examples in figures 19 and 20 use the data for the same example as that in figure 17. The use of relative concentrations and dilution factors may at first appear complex but, once understood, will simplify computational procedures and laboratory work involved in such measurements.

Relative concentrations based on stock solutions

All measurements start with the stock dye solution as supplied by the manufacturer, having a concentration C_s (see fig. 19). For

CONCEPTUAL DIAGRAM



EXAMPLE COMPUTATION

$$\frac{C}{C_4} = \frac{S_G C_s D_C}{S_G C_s D_T} = \frac{D_C}{D_T} \quad C = C_4 \frac{D_C}{D_T}$$

Equation 9: $Q = 5.89 \times 10^{-7} \ q \frac{C}{\bar{c}}$

By substitution:

$$Q = 5.89 \times 10^{-7} \ q \left[\frac{C_4}{\bar{c}} \times \frac{D_C}{D_T} \right]$$

Since samples of C_4 and \bar{c} are to be read on the same scale

$C_4 = kR$ and $\bar{c} = kr$, where k is a scale constant.

$$\text{Therefore } Q = 5.89 \times 10^{-7} \ q \left[\frac{R}{r} \times \frac{D_C}{D_T} \right]$$

From data in figure 17 and dilution factors in table 4, line 3, and table 2B, line 9:

$$Q = 5.89 \times 10^{-7} (31.2) \left[\frac{45.5}{48.6} \times \frac{10^{-1}}{4.20 \times 10^{-8}} \right] = 41.0 \text{ ft}^3/\text{s}$$

Figure 19.—Conceptual diagram and example of computations using relative concentrations and dilution factors originating with the manufacturer's stock dye solution.

rhodamine WT, this is 20 percent by weight. As seen in figure 19, C_s is usually reduced to some lesser concentration C for injection into the stream. As seen in column 4 of tables 3 and 4, based on the volumes of water and dye mixed, a precise dilution factor, D_C , may be computed using equation 7:

$$C_n = C_i S_G \left[\frac{V_d}{V_w + V_d} \right] = C_s S_G D_C$$

where

$$C_i = C_s \text{ and } \left[\frac{V_d}{V_w + V_d} \right] = D_C .$$

In the previous examples, a stock solution C_s of rhodamine WT 20 percent was reduced before injection to a C of $2.38 \times 10^7 \mu\text{g/L}$ by combining 100 mL of dye with 900 mL of water (see table 4, line 4). Thus C_s was diluted by a factor, D_C , of 100/1,000 or 10^{-1} . The concentration in the stream \bar{c} is then a function of the rate of injection of C and of its dilution by Q .

In preparing laboratory standards, a sample of C_s is reduced until a final concentration, C_4 , is reached after a four-step serial dilution (see fig. 19). Thus, by this serial-dilution process, the standard C_4 , the one tailored to closely match that expected in the stream, \bar{c} , is prepared from C_s as

$$C_4 = S_G C_s D_T$$

where D_T is the total dilution required and the product of the individual dilutions. In table 2B, the total dilution factor after four dilutions is shown in column 5. The total dilution factor for the $10\text{-}\mu\text{g/L}$ standard used in the previous example is

$$D_T = D_1 \times D_2 \times D_3 \times D_4$$

$$D_T =$$

$$\left[\frac{30}{50+3792} \right] \left[\frac{20}{20+3500} \right] \left[\frac{20}{20+3500} \right] \left[\frac{50}{50+450} \right]$$

$$D_T = 4.20 \times 10^{-8}$$

and the final concentration, C_4 , starting with rhodamine WT 20 percent is

$$C_4 = C_s S_G D_T = 20 \times 10^7 \times 1.19 \times 4.20 \times 10^{-8} \\ = 10 \mu\text{g/L}$$

as desired.

Using the net dial readings R and \bar{r} in place of C and \bar{c} in equation 9 and the appropriate dilution factors, the discharge may now be computed as

$$Q = 5.89 \times 10^{-7} q \left[\frac{R}{\bar{r}} \times \frac{D_C}{D_T} \right] \quad (10)$$

and, using the example data provided in figure 17 and tables 2B and 4, the discharge may be computed as

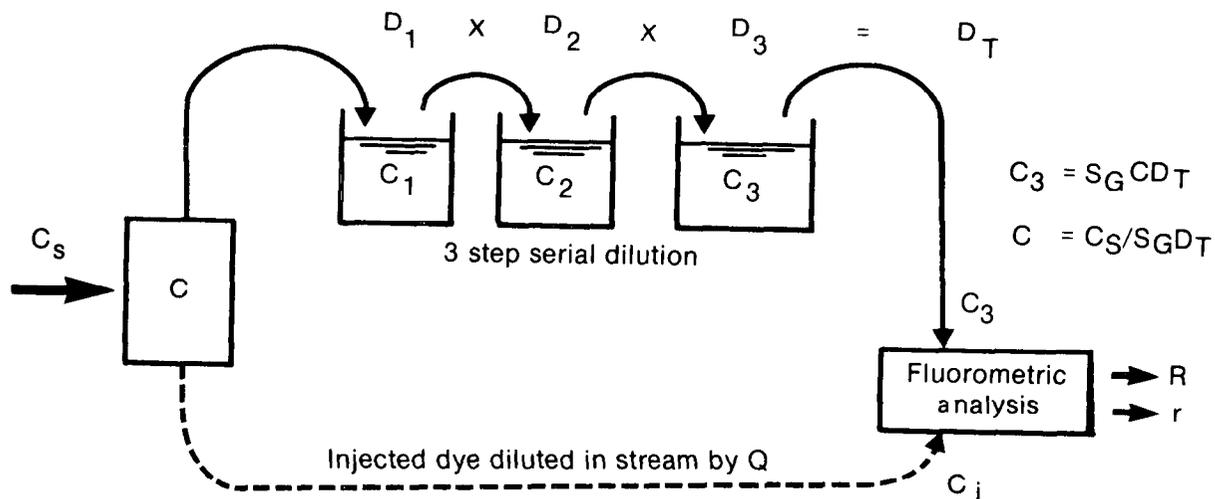
$$Q = 5.89 \times 10^{-7} \times 31.2 \left[\frac{45.5}{48.6} \times \frac{10^{-1}}{4.20 \times 10^{-8}} \right] \\ = 41.0 \text{ ft}^3/\text{s} .$$

Note that if C is selected as a concentration shown in column 1 or 5 of table 3 or 4, respectively, and if C_4 is chosen as one of those in column 4 in table 2B, the determination of the discharge is essentially a matter of determining the ratio of the fluorometer dial readings for the standard and the stream samples. The dilution factors, D_C and D_T , may be taken from tables 2B, 3, and 4 or any convenient tables the user might wish to develop. Certain precautionary measures must be taken in using this technique. All measurements and computations start with C_s and assume that the manufacturer's solution is well mixed and the same whenever quantities are taken from the container of a particular dye lot.

Relative concentrations based on injected solutions

A schematic diagram and an example of computations of discharge, using relative concentrations and dilution factors starting with the concentration, C , of the dye injected rather than C_s , are shown in figure 20. This method has the advantage of avoiding errors that might occur if the manufacturer's solution was not mixed each time dye was extracted, or if an

CONCEPTUAL DIAGRAM



COMPUTATIONS

Equation: $Q = 5.89 \times 10^{-7} q \frac{C}{\bar{c}}$

By substitution:

$$Q = 5.89 \times 10^{-7} q \left[\frac{C_3}{\bar{c}} \times \frac{1}{S_G D_T} \right]$$

Since samples of C_3 and \bar{c} are to be read on the same scale.

$C_3 = kR$ and $\bar{c} = k\bar{r}$, where k is a scale constant.

Therefore

$$Q = 5.89 \times 10^{-7} q \left[\frac{R}{\bar{r}} \times \frac{1}{S_G D_T} \right]$$

Note: S_G is the specific gravity of C , not of rhodamine WT 20-percent.

If C is $1 \times 10^7 \mu\text{g/L}$ or less, S_G will be less than 1.01 and can be ignored.

EXAMPLE: As in the example in figure 17: $C = 2.38 \times 10^7 \mu\text{g/L}$
 $q = 31.2 \text{ mL/min}$
 $\bar{r} = 48.6$ net dial reading on
 scale III of the fluorometer
 $S_G = 1.02$

Using table 5A, prepare a standard, $C_3 = 10.34 \mu\text{g/L}$ (line 3c)
 This standard yielded a net dial reading on scale III of 47.0
 Thus the discharge is

$$Q = 5.89 \times 10^{-7} (31.2) \left[\frac{47.0}{48.6} \times \frac{1}{(1.02) (4.34 \times 10^{-7})} \right] = 40.1 \text{ ft}^3 \text{ 3/s}$$

Figure 20.—Conceptual diagram and example of computations using relative concentrations and dilution factors originating with the injected solution.

Table 5.—Convenient three-step serial-dilution combinations for the preparation of selected standards for selected dye-injection solutions; prepackaged solutions.

Injection solution C in $\mu\text{g/L} \times 10^7$	Dilutions						Resulting conc. C_3 in $\mu\text{g/L}$	Resulting dilution factor, $D_T \times 10^{-7}$
	First		Second		Third			
(1)	V_d	V_w	V_d	V_w	V_d	V_w	(8)	(9)
1a 0.60	50	3000	20	2000	20	3000	6.45	10.75
b	50	3000	20	2000	20	2500	7.73	12.88
c	50	3000	20	2000	20	2000	9.64	16.07
d	50	3000	20	2000	20	1500	12.82	21.36
2a 1.19	50	3000	20	3000	20	3500	7.34	6.17
b	50	3000	20	3000	20	3000	8.56	7.19
c	50	3000	20	3000	20	2500	10.25	8.62
d	50	3000	20	3000	20	2000	12.79	10.75
3a 2.38	25	3000	20	3000	20	3500	7.40	3.11
b	25	3000	20	3000	20	3000	8.63	3.62
c	25	3000	20	3000	20	2500	10.34	4.34
d	25	3000	20	3000	20	2000	12.90	5.42
4a 4.76	20	3000	20	3500	10	2000	8.91	1.87
b	20	3000	20	3500	20	3500	10.18	2.14
c	20	3000	20	3500	20	3000	11.86	2.49
d	20	3000	20	3500	20	2750	12.93	2.72
5a 5.95	20	3500	20	3500	10	2500	7.65	1.29
b	20	3500	20	3500	10	2000	9.56	1.61
c	20	3500	20	3500	10	1750	10.91	1.83
d	20	3500	20	3500	10	1500	12.72	2.14
6a 7.14	20	3500	20	3500	10	3000	7.66	1.07
b	20	3500	20	3500	10	2500	9.18	1.29
c	20	3500	20	3500	10	2000	11.47	1.61
d	20	3500	20	3500	10	1750	13.10	1.83
7a 11.90	20	3500	10	3000	10	3000	7.46	0.627
b	20	3500	10	3000	10	2250	9.94	0.835
c	20	3500	10	3000	10	2000	11.18	0.939
d	20	3500	10	3000	10	1750	12.76	1.073

error was made in measuring the quantities of dye and water in preparing the injection solution. Thus the stream discharge could be determined if the absolute value of C was unknown but a sample of C was retained from each test. This is good practice regardless of the method of analysis. The fact that C/\bar{c} is a ratio allows the comparison and measurement of \bar{c} relative to C using dial readings and measured dilutions. Unlike the previous method in which a working solution of $100 \mu\text{g/L}$ is prepared and retained for each dye lot (and, hence, only a single additional dilution, D_4 , is required for a measurement), a three-step serial dilution of C must be done for each test where a different injection solution is used. This emphasizes the

advantages of preparing a bulk quantity of dye solution of a single concentration, such as one of those in table 3, for a series of discharge measurements. The same would be true for the use of prepackaged dye quantities (such as in table 4); selection of a single dye concentration, C , for a series of measurements always lessens the laboratory work in preparing standards.

By starting with the injected solution, C , only a three-step serial dilution need be prepared to obtain a standard about equal to that expected in the stream for \bar{c} . Tables 5 and 6 provide convenient combinations of dye and water to yield a range of standards from about 7 to $13 \mu\text{g/L}$ (column 8), which should be the range obtained in the field samples. Column 9

Table 6.—Convenient three-step serial-dilution combinations for the preparation of selected standards for selected dye-injection solutions; bulk solutions.

Injection solution C in $\mu\text{g/L} \times 10^7$	Dilutions						Resulting conc. C_3 in $\mu\text{g/L}$	Resulting dilution factor, $D_T \times 10^{-7}$
	First		Second		Third			
(1)	V_d	V_w	V_d	V_w	V_d	V_w	(8)	(9)
8a 0.1	50	3000	50	1000	20	2000	7.73	77.29
b	50	3000	50	1000	20	1750	8.82	88.21
c	50	3000	50	1000	20	1500	10.27	102.71
d	50	3000	50	1000	20	1000	15.31	153.07
9a 0.2	50	3000	50	2000	20	2000	7.92	39.59
b	50	3000	50	2000	20	1750	9.04	45.18
c	50	3000	50	2000	20	1500	10.52	52.61
d	50	3000	50	2000	20	1250	12.59	62.97
10a 0.4	50	3000	50	3000	20	1500	14.14	35.36
b	50	3000	50	3000	20	1750	12.15	30.37
c	50	3000	50	3000	20	2000	10.64	26.61
d	50	3000	50	3000	20	2500	8.53	21.33
11a 0.6	50	3000	20	2000	20	3000	6.45	10.75
b	50	3000	20	2000	20	2500	7.73	12.88
c	50	3000	20	2000	20	2000	9.64	16.07
d	50	3000	20	2000	20	1500	12.82	21.36
12a 0.8	50	3000	20	2000	20	3000	8.60	10.75
b	50	3000	20	2000	20	2750	9.38	11.72
c	50	3000	20	2000	20	2500	10.31	12.88
d	50	3000	20	2000	20	2000	12.86	16.07
13a 1.0	50	3500	20	3000	20	2500	7.40	7.40
b	50	3500	20	3000	20	2000	9.24	9.24
c	50	3500	20	3000	20	1750	10.54	10.54
d	50	3500	20	3000	20	1500	12.27	12.27
14a 1.5	50	3500	20	3000	20	3500	7.95	5.30
b	50	3500	20	3000	20	3000	9.27	6.18
c	50	3500	20	3000	20	2500	11.10	7.40
d	50	3500	20	3000	20	2000	13.85	9.24
15a 2.0	50	3500	20	3500	10	2000	7.96	3.98
b	50	3500	20	3500	20	3500	9.09	4.55
c	50	3500	20	3500	20	3000	10.60	5.30
d	50	3500	20	3500	20	2500	12.70	6.35
16a 3.0	25	3500	20	3000	20	3500	8.01	2.67
b	25	3500	20	3000	20	3000	9.33	3.11
c	25	3500	20	3000	20	2500	11.18	3.73
d	25	3500	20	3000	20	2000	13.95	4.65
17a 4.0	25	3500	20	3500	10	2000	8.02	2.00
b	25	3500	20	3500	20	3500	9.16	2.29
c	25	3500	20	3500	20	3000	10.67	2.67
d	25	3500	20	3500	20	2500	12.79	3.20
18a 5.0	20	3500	20	3500	10	2000	8.03	1.61
b	20	3500	20	3500	20	3500	9.17	1.83
c	20	3500	20	3500	20	3000	10.69	2.14
d	20	3500	20	3500	20	2500	12.81	2.56
19a 10.0	20	3500	20	3500	10	3750	8.59	0.859
b	20	3500	20	3500	10	3500	9.20	0.920
c	20	3500	20	3500	10	3000	10.73	1.07
d	20	3500	20	3500	10	2500	12.86	1.29

in tables 5 and 6 provides the total dilution factors, D_T , for whatever concentration is used. Following the diagram in figure 20, it can be seen that the standard, C_3 , is equal to $S_G CD_T$. Thus, $C = C_3/S_G D_T$ and the discharge equation, substituting net dial readings for concentrations, is

$$Q = 5.89 \times 10^{-7} q \left[\frac{R}{\bar{r}} \times \frac{1}{S_G D_T} \right] \quad (11)$$

As an example, the same test conditions as those previously used and presented in figure 17 are assumed, except that now a three-step serial dilution of C must be performed. Referring to table 5, a convenient serial dilution combination for the case where $C = 2.38 \times 10^7 \mu\text{g/L}$ is found on line 3c (for $C_3 = 10.34$), since the injection rate for the estimated stream discharge was chosen to produce a plateau concentration of about $10 \mu\text{g/L}$ (fig. 9). A standard, C_3 , is analyzed on the fluorometer on scale III at the same time the field samples are analyzed. This sample gives a dial reading of 47.2 for a net of 47.0 (the stream background was 0.2). Thus for the stream samples taken at 25 minutes after the start of injection and having an average dial reading, \bar{r} , of 48.6, the discharge may be computed as

$$Q = 5.89 \times 10^{-7} \times 31.2 \left[\frac{47.0}{48.6} \times \frac{1}{(1.02)4.34 \times 10^{-7}} \right] \\ = 40.1 \text{ ft}^3/\text{s}$$

The dilution factor, D_T , was picked from column 9, line 3c of table 5 for the standard chosen. Note that in tables 5 and 6, the first two dilutions for any one concentration, C , are the same. Thus the second dilution may be treated as a working solution. If the $10.34\text{-}\mu\text{g/L}$ standard was not close to that experienced in the stream (R did not approximately equal \bar{r}), another standard could be prepared quickly, starting with the second dilution.

This method must be considered the most accurate of those presented, because only a three-step serial dilution of C is involved. Any error in preparing the injection solution is eliminated, and fluorometer dial readings are used directly without recourse to calibration curves.

Appendix B contains a form for recording field sampling, fluorometer analysis, and calculation of discharge using the constant-injection method. It incorporates the use of relative concentrations, dilution factors, and discharge weighting. Provision is made for sampling as many as five locations laterally across a flow section; however, on narrow well-confined flows, three sampling locations are usually sufficient unless the adequacy of mixing is suspect. Appendix B also contains a completed form using the data for the example measurements illustrated in figures 17 and 19 where relative concentrations are used.

Mixing Length

The mixing length equation, equation 4, can be rewritten as

$$L_m = K \frac{vB^2}{E_z} \quad (12)$$

where the variables are as previously defined, except that K is a variable with a value dependent on the degree of mixing, the location of injection and the number of injections. The degree of mixing is a measure of the degree to which a tracer is mixed in the flow; the higher the percentage, the more nearly are the concentrations the same laterally. Appendix A shows the recommended method of computing percentage of mixing.

The value of K of 0.1 given in equation 4 is for 95-percent mixing with a single-center injection at midstream. Coefficients, K , for other conditions are given in table 7.

The effect of injecting tracer at n points, where each injection is at the center of flow of each n equal-flow segment, is that the tracer has to mix throughout an equivalent width of about $(1/n)B$. Because B is squared in the mixing length equation, the value of K for a single-point injection is modified by the factor $(1/n)^2$.

Injection at the side of a channel is equivalent to an injection into a stream with a width of $2B$. Because B is squared, the coefficient K must be increased by a factor of 4.

Table 7.—Values for coefficient, K, for different degrees of mixing and numbers and location of injection

Number and location of injection points	Coefficient, K		
	Percentage mixing		
	90	95	98
One center injection	0.070	0.100	0.140
Two injection points ^a	0.018	0.025	0.035
Three injection points ^b	0.008	0.011	0.016
One side injection point	0.280	0.400	0.560

^a For an injection made at the center of each half of flow.

^b For an injection made at the center of each third of flow.

Where the mixing length must be shortened, it is best to inject at two or three points in the cross section. A line injection is not recommended, with the possible exception of canals where very uniform flow exists across the section (see fig. 21). If the injection rate from a line source is not proportional to the flow at each point across the section, the use of a line source may actually increase the mixing distance.

Equation 12 is for an injection in the center of each flow segment and for a fairly uniform channel. If the injection is not in the center of flow, the theoretical mixing distance will be increased. The equation for mixing provides only an estimate of mixing length. Experience at a site may indicate a very different mixing distance. From a practical and theoretical standpoint, the narrower and deeper the flow, the shorter the channel length required for mixing. Thus, mixing length may change with stage; a flood flow that fills the channel may require a significantly greater channel length than a lower flow because of higher velocity and greater width. However, the effect of increased depth partly offsets this difference.

A common fallacy is that shallow, turbulent whitewater streams produce rapid mixing. Turbulence is effective in yielding rapid vertical mixing, but unless there is substantial depth, lateral dispersion occurs gradually and the mixing length may be quite long.



Figure 21.—Multiple dye injection in the approach to a Parshall flume; not visible is dye-injection pump, which preinjects dye into a hose line from a domestic water supply. Eight samples taken across the channel about 10 ft below the hydraulic jump in the bottom of the picture yielded good results.

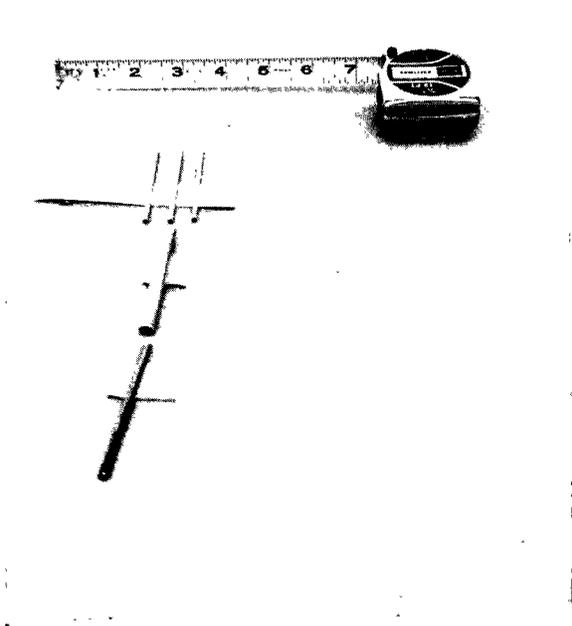


Figure 22.—Tubing used in the fabrication of a one-to-three manifold for multiple injection of dye.

Mixing in pool-and-riffle streams sometimes occurs in relatively short distances, although the time to reach an equilibrium plateau (in the case of a constant-rate injection) should not be underestimated. Streams with deadwater areas or shallow side flows should be avoided; such conditions produce the long, attenuated tracer clouds requiring long injection times.

For further information on mixing, see Yotsukura and Cobb (1972).

Injection Schemes

In most cases, a single-point injection of dye is used. The injection line may ordinarily be placed over the flow with no difficulty, as shown in figure 16. In some instances, however, excessive stream width or depth and flow velocities or the lack of bridges may make proper placing of the injection line difficult. If the station is one where dye-dilution measurements are to be made repeatedly, it may be worthwhile to install a permanent apparatus. Figure 13 shows a 5-gal can mariotte vessel positioned over a stream by use of a light cable and trolley. In this instance, the injection rate is measured volumetrically several times with a 100-mL graduated cylinder and stopwatch

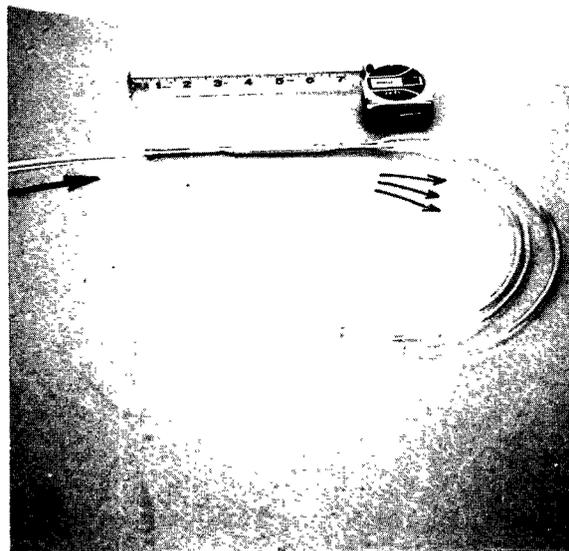


Figure 23.—Fabricated manifold for three-point multiple dye injection; the three lengths of tubing should be equal to produce three equal injection rates.

before allowing the mariotte vessel to roll out over the stream. A light rope is secured to the vessel for recovery.

Where a pump is used, the injection line may be permanently secured over the stream by attaching it to a cableway, running it through a pipe, or attaching it to the underside of walkways and bridges. In each case, the pump is attached to the line for injection and, upon completion of the measurement, a solution of alcohol and water is flushed through both the pump and line.

Multiple injection may be desirable in wide channels with shallow flow or where rapidly varying flow is to be measured and mixing lengths are to be minimized. If a multiple-injection scheme is needed, multiple lines may be permanently located to distribute the dye injection in approximate proportion to the flow. Figures 22 and 23 show the fabrication of a one-to-three tube manifold device to permit a three-point injection from a single injection source. The manifold consists of three equal lengths of 1/16-in-diameter thin brass tubing nested and soldered inside a 3/8-in section of

brass tubing. A 1/8-in piece of copper tubing is soldered into the other end of the 3/8-in tubing to receive the discharge tube from a pump. Three plastic lines of equal length are led to the stream for positioning over the flow.

Another approach involves the preinjection of the dye into a larger line leading to a sprinkler line. In figure 21, a sprinkler line consisting of a perforated rigid pipe is placed over the approach of a Parshall flume. Unchlorinated tap water from a domestic hose line is fed to the pipe; dye has been preinjected into the hose, permitting it to mix with the larger flow of tap water before discharging through the sprinkler. An injection pump or chemical-feed device capable of injecting under pressure is necessary. The injection rate is calculated by measuring volume or weight change with time, as discussed previously.

A sprinkler-type system across the flow should not be used to shorten the mixing distance unless the flow is fairly evenly distributed across the channel. If the flow is not evenly distributed, use of a sprinkler system may actually increase the mixing distance.

Dye Losses

The section on theory discusses the principles involved in performing slug and constant-rate dye-dilution discharge measurements. Experience shows that measurement errors are often falsely attributed to dye losses. As can be seen from equation 9, a dye loss would produce a low value for \bar{c} , and the computed discharge would be high. In many instances, failure to inject long enough or sampling too soon results in low values of c , particularly along the stream banks (see fig. 3). Using a sufficiently long injection and waiting long enough to sample will permit the slowly developing plateaus along the banks to stabilize. Dye losses do occur, but the above cautions need to be emphasized.

Real dye losses occur where excessively long stream reaches are involved; where there is finely suspended sediment, particularly clay or flocculent organic particles; and where chemicals like chlorine oxidize or quench the dye. To avoid quenching, measurements should not normally be made on streams having a pH less than five (Smart and Laidlaw, 1977). Where a

low pH does exist, the use of pontacyl pink dye is advisable. Dye losses have also been noted on very turbulent streams. The high oxygen levels may oxidize the dye in the same way that chlorine does.

If the stream to be measured may contain substances that can cause a dye loss, a gallon of the stream water should be obtained for laboratory use in preparing a standard to be compared with a duplicate one prepared from distilled water. The laboratory work must be done promptly, because the chemistry of the river water may change if it is stored too long. A stream-water standard may be prepared in the field at the time of the dye test if a 100- $\mu\text{g/L}$ working solution for the dye lot is available.

The preparation of a complete set of standards from stream water with the objective of cancelling any reactions that may be taking place in the stream is to be discouraged. Normally, it is difficult to replicate reactions between dye and stream water in a bottle. Standards should provide absolute values without the uncertainty inherent in using water that may produce unpredictable results.

Adhesion of the dye to particle surfaces does not appear to take place immediately. In special situations, dye losses from adhesion on fine sediment have been avoided by immediate filtration of the samples to minimize the exposure of the dye to particles.

Tests by Rathbun (USGS, written communication, 1983) indicate that significant losses of rhodamine WT may occur where diluted concentrations are exposed to direct sunlight for several hours. Thus, excessive reach lengths, like those used in time-of-travel studies, may result in significant photochemical decay of rhodamine WT dye. Most exposure times for dye-dilution discharge measurements are short enough that photochemical decay losses are not a problem.

Special Measurements

Beneath ice

The measurement of discharge in ice-covered streams by conventional current-meter methods is laborious and often of questionable accu-

racy. On small streams, the dye-dilution method of measuring discharge is particularly useful under such conditions because it does not depend on measurement of either the flow area or velocity.

Site preparations

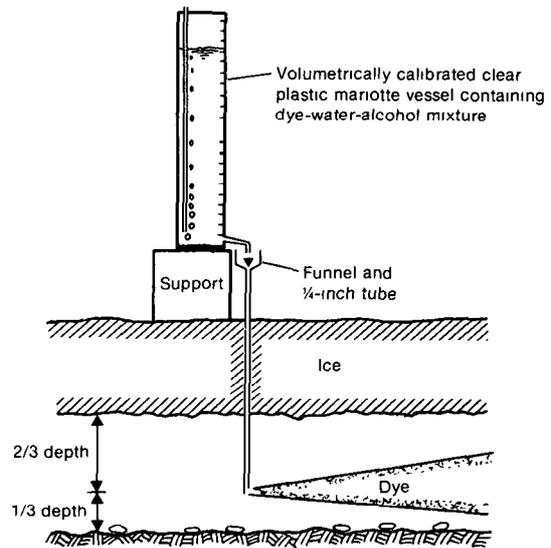
For most small streams, a single, midchannel dye injection is sufficient. Preparations, including selection of the reach length, are the same as the criteria discussed in the earlier section for the slug-injection test. Equation 4 can be used to estimate the mixing length; the deeper and narrower the channel, the better.

Unlike open-channel, current-meter flow measurements, selection of the channel reach is not limited by availability of bridges; a firm ice cover will permit ready access to the stream for placing equipment and selecting the injection and sampling sites. Unlike current-meter measurements beneath ice, only very small holes need be drilled through the ice for injection and sampling. A single 1/2-in-diameter hole through the ice in the centroid of the flow is used for the injection. It may be necessary to drill several exploratory holes to find the main flow. It is well to select a measurement reach before freezeup. The injection line may be supported beneath the ice by inserting it inside a 3/8-in-diameter metal or plastic tube. This tube will usually vibrate when projected into the main flow and this can be used as a guide in selecting the centroid of flow. As shown in figure 24, the dye-injection line should be positioned well beneath the ice but clear of the streambed. Slush ice conditions immediately downstream from the injection should be avoided.

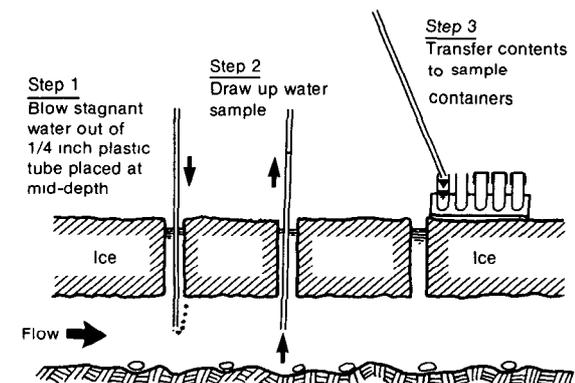
Three to five small holes should be drilled across the channel at the section chosen for sampling. An ordinary carpenter's brace and long auger wood bit may be used to cut the small holes through the ice. The spiral helps to remove ice cuttings (see figs. 25 and 26). Grinding off the small starting tip of the wood bit makes it more suitable as an ice auger. Because of the smallness of the holes, they may have to be marked with rocks or sticks to be readily located when needed. Too small a hole may cause drill freezeup, and the hole itself is apt to freeze back before it can be used.

Tracer preparation and injection

For a constant-injection test, the amount of tracer needed should be calculated from the estimated discharge and the graph in figure 9. The constant-injection type of test is recommended for measurements beneath ice. Rhodamine WT dye 20-percent solution as received from the manufacturer has a specific gravity of approximately 1.19. Normally, this solution is mixed with water in amounts such as shown in tables 3 and 4. When performing dye-dilution measurements under freezing conditions, the addition of antifreezing agents such as methanol alcohol or glycol is mandatory if the



(A) Injection system for measurement beneath ice



(B) Method of "pipetting" water samples from under the ice

Figure 24.—Dye-dilution discharge measurement under ice cover.

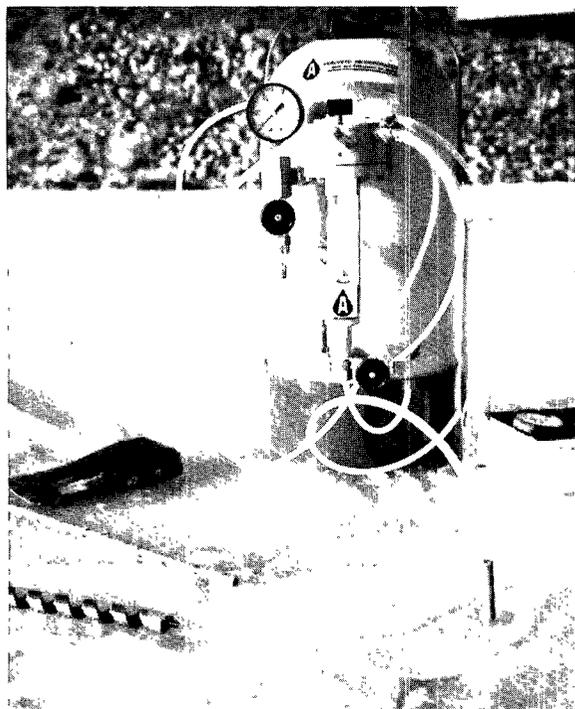


Figure 25.—Dye injection under ice cover, using a chemical-feed tank. An auger is used to drill holes through the ice and a tube is used to draw background samples from under the ice before injection.

small injection rates used are not to be interrupted or altered by solution freezeup. Automobile-type antifreeze should not be used because it frequently contains sealers and rust inhibitors that may clog the injection apparatus. Methanol alcohol is recommended for addition to dye-water solutions because it has a specific gravity of 0.80 and, if added in equal proportions to the dye, will yield essentially a neutrally buoyant mixture. Figure 27 shows the amount of alcohol to be used for a given temperature. The sum of the water and alcohol should equal the volumes of water shown in column 2 of table 3 or the total volume of water-dye mixture in column 3, table 4. Unless a neutrally buoyant solution is used or the injection solution is below 2 percent concentration, the injection rate should not be determined by weighing; volumetric-displacement methods should be used, if possible.



Figure 26.—Sampling under ice cover as part of a dye-dilution discharge measurement; an auger is used to drill three to five holes through the ice across the channel at a suitable distance downstream from the injection, and a tube is used to draw samples.

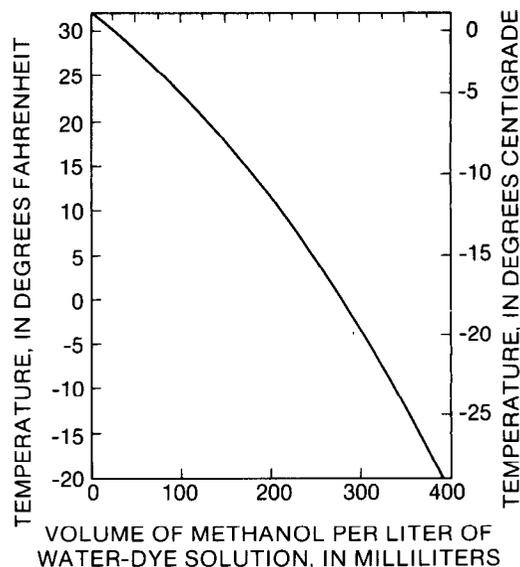


Figure 27.—Proportions of methanol alcohol and water-dye solution needed to prevent freezing of mixtures at selected temperatures.

Dye injection using the constant-rate pump for withdrawing the dye solution from a graduated cylinder is recommended. The backpack enclosure shown in figures 11 and 12 is suggested, for it protects the equipment from freezing winds. As shown in figure 24A, a mariotte vessel may be substituted for the constant-rate pump by using a funnel and tube arrangement. All equipment should be thoroughly flushed with an alcohol-water solution upon completion of the measurement.

Sampling

It is important that the water sample be taken below the ice in flowing water. To obtain a representative sample, a rigid plastic tube with a 1/4-in inside diameter is inserted through the ice hole until the tube is below the ice but not hitting the streambed. If the flow is swift, the plastic tube will vibrate in the flow. This and the depth at each sampling hole can be a guide in estimating the proportion of flow to be allotted each sample. Draw freshwater up into the tube, taking care not to swallow any, and immediately blow it back out. That washes out previous samplings and removes any stagnant water that may enter the tube as it is first lowered through the hole. Next, draw or pipet in a freshwater sample, hold it in the tube by placing a finger over the top, and withdraw the tube from the hole, transferring the contents to suitable containers. Samples of about 15 to 20 mL are needed for laboratory analysis. In addition to the river samples taken during the actual dye tests, water samples for background analyses should be taken before the injection.

Analysis

The samples should be protected from freezing and returned to the laboratory for the usual analysis. Analyses and computations are the same as those described for other dye-dilution, constant-injection discharge measurements.

Canals and pipes

The data shown in figures 8 and 17 for a small, moderately sloping stream with a depth-

to-width ratio of 1 to 20 and a mixing length of 700 ft are typical of a natural stream. Using those data and referring to figure 2, the total time of passage of a slug-response curve, T_D , was 7 minutes, and a stable plateau concentration was achieved 12 minutes after the start of injection. Had greater mixing distances been employed or had the natural channel contained more slack-water zones, a longer period of injection would have been necessary before a plateau was reached.

When a tracer is slug injected into manmade canals or pipes, the resulting response curves are much less elongated than those depicted in figures 1, 2, and 3 because of the lack of slack-water areas. Consequently, the plateau in response to a constant-rate tracer injection will also be reached rapidly in lined canals and pipes. Figure 28 shows time-concentration data obtained from a 4-minute-duration, constant-rate injection in a large irrigation canal in Colorado. Note that, although the distance was more than a mile, a plateau was reached within 3 minutes, 16 minutes after the start of injection. The mean velocity in this canal during the measurement was more than 7 ft/s; as seen in equation 4, the mixing length required is directly proportional to the mean velocity. Thus, in such structures, the time for mixing may be brief but the distance large.

Similar results may be expected in pipes and other manmade conduits where velocities are high and pools and riffles nonexistent. If adequate mixing lengths are physically available, excellent dye-dilution measurements can be made in such structures.

Measurement During Unsteady Flow

Concepts

According to the constant-rate injection formula for discharge, equation 2, if tracer is injected continuously at a constant rate into a flow varying as shown by the solid line in fig-

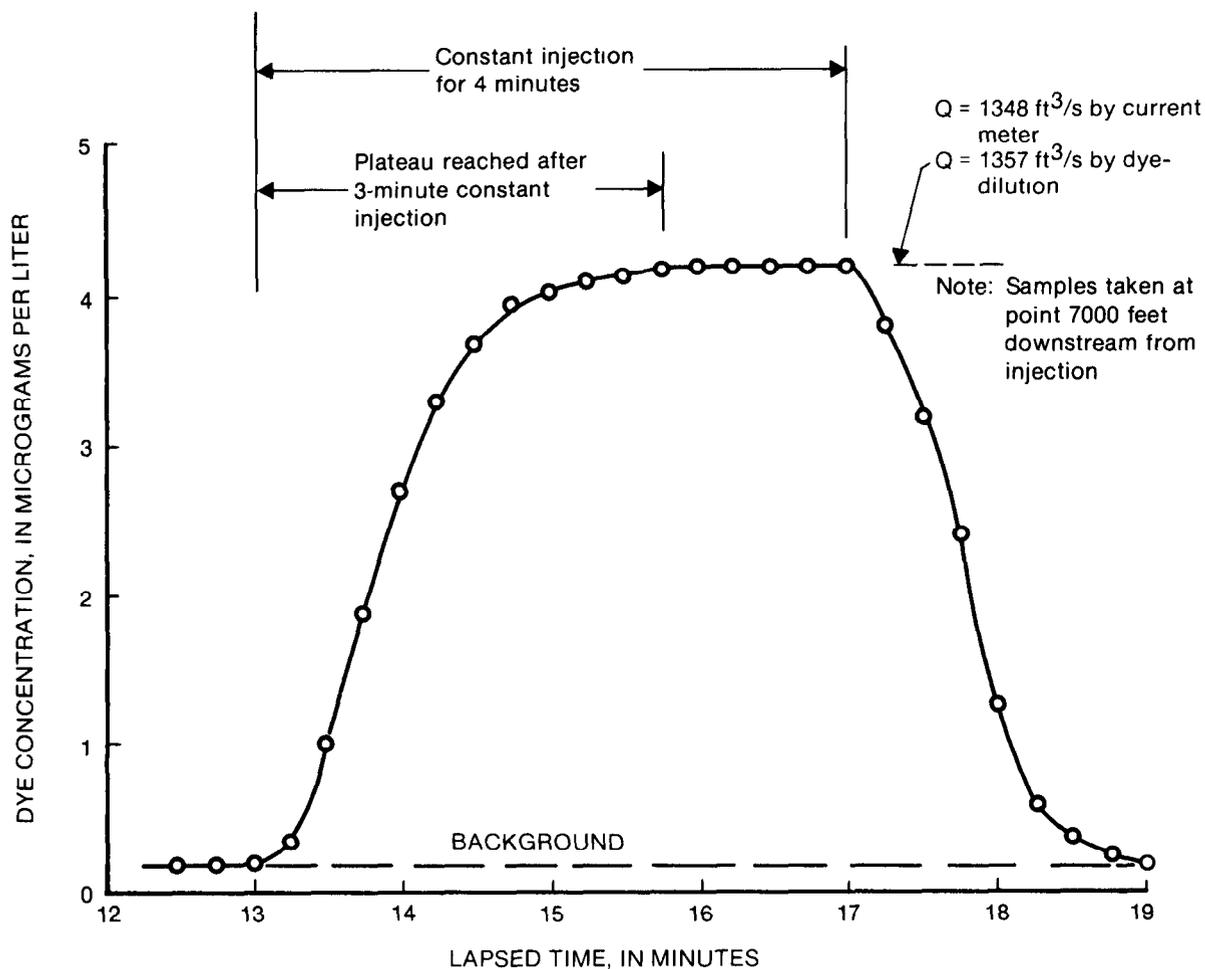


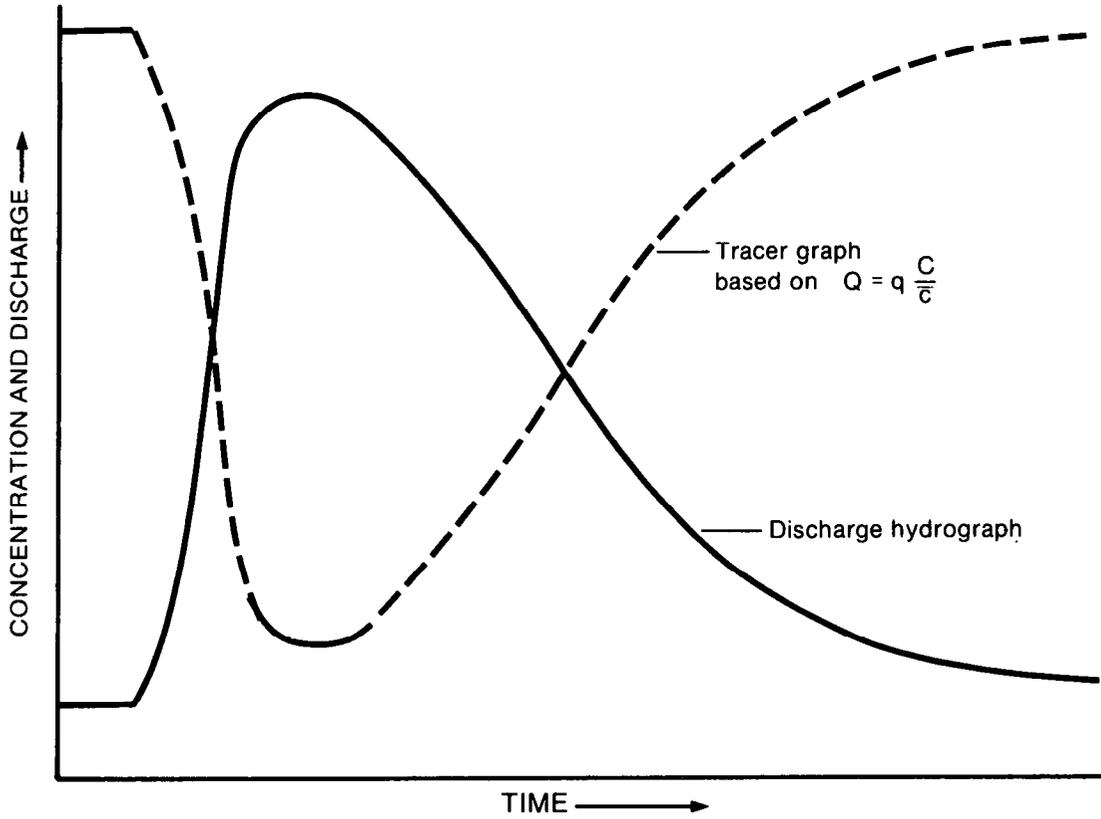
Figure 28.—Rapid measurement of discharge in a lined canal by dye dilution, Charles Hanson Canal, Colorado.

ure 29A, the inverse tracer concentration graph shown as a dashed line results. Theoretically, any sample taken (assuming adequate mixing) during such an unsteady flow would be a measure of the discharge at that instant. Thus, interesting possibilities exist for the accurate measurement of unsteady or rapidly varying flow.

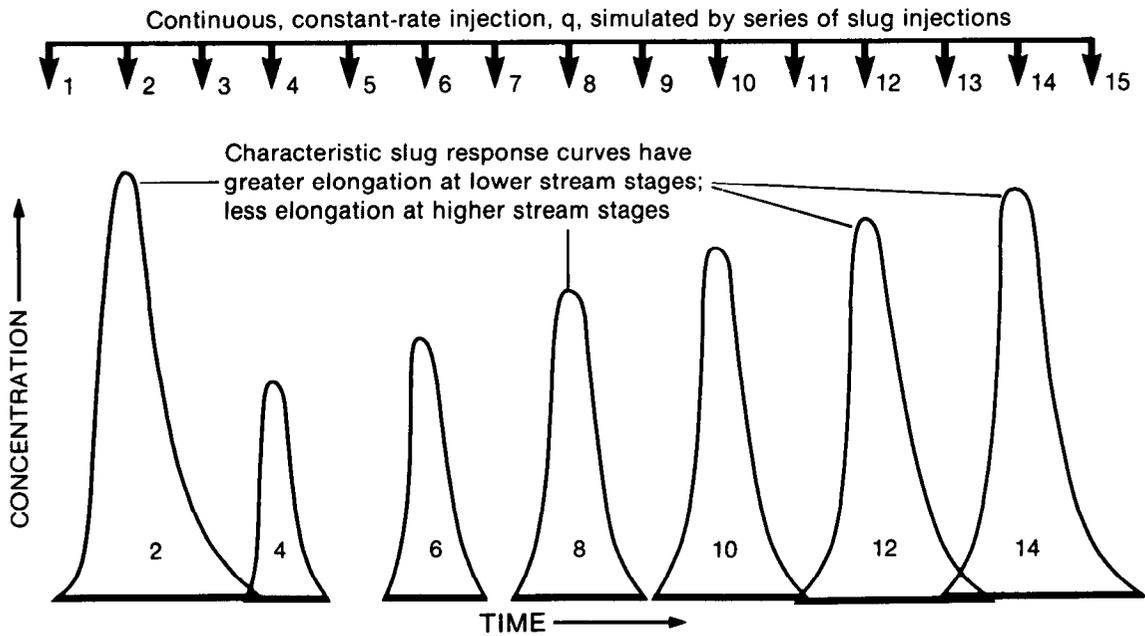
Unfortunately, certain factors limit the degree of unsteadiness that can be measured practically by dilution techniques. Fortunately, field tests show that the method can be used and equation 9 applied for rather high degrees of unsteady flow. Additional laboratory and field tests are in progress to attempt to

define the conditions and limits of use of the constant-rate injection method for the measurement of unsteady flow.

For the theoretically correct measurement of unsteady flow using a continuous injection, a given element of tracer would have to mix instantaneously in the flow element into which it was injected. That is, the longitudinal dispersion that produced the ever-elongating tracer response curves shown in figure 1 could not exist. But of course it does exist; and, in steady flow, the plateau is produced by the summation of the superposed slug-response curves, as shown in figure 2, which are shaped by longitudinal dispersion. The greater the elongation of



A. Tracer hydrographs produced by constant-rate injection into unsteady discharge



B. Conceptualized slug responses that occur at different discharges; every other response curve is shown.

Figure 29.—Constant-rate injection into unsteady flow.

the characteristic slug-response curves, the greater the probable error in a constant-injection dye-dilution measurement in unsteady flow.

A precise theoretical analysis of a continuous, constant-rate tracer injection into unsteady flow has not yet been accomplished. Some idea of the factors involved can be ascertained, conceptually at least, by using the superposition principle discussed in the section on Theory, in which the plateau resulting from a constant injection of tracer into steady flow is simulated by superposing the characteristic slug-response curves (fig. 2). In figure 29B, the characteristic slug-response curves are superposed but increased or decreased in concentration to reflect the dilution that would occur if each slug mass were mixed into its respective element of flow. Furthermore, as flow depth and velocities increase at the higher stages, longitudinal dispersion typically decreases; that is, slug-response curves become shorter in time of passage and, thus, favorable to an accurate dilution measurement of unsteady flow. At the same time, it is probable that the mixing length required to measure the discharge hydrograph shown in figure 29A would be the longest (mixing length would change primarily due to change in depth and velocity) at the higher discharges until bankfull stage was reached. Thus, the reach length chosen would have to be greater than necessary for the lower discharges and the characteristic response curve for the lower stages elongated excessively over that needed for an accurate dilution measurement. This occurs particularly when pool-and-riffle rather than channel-control flow conditions exist. The characteristic response curves shown in figure 29B have, therefore, been altered from the steady-state shape to reflect dilution and change in longitudinal dispersion with stage and discharge. Figure 29 provides conceptually the following insights into what is apt to happen with the continuous constant-rate injection into an unsteady flow and where the best accuracy for discharge measurements is likely.

1. Greatest errors are apt to occur on the rapidly changing rising limb of the discharge hydrograph.

2. Lesser longitudinal dispersion at higher stages just past the peak and on the recession will result in the most accurate results during this part of the hydrograph.

Results

Constant-rate, dye-dilution measurement tests of unsteady flow have been successfully performed by F. A. Kilpatrick, by Katz and Fischer (1983), and by M. D. Duerk (U.S. Geological Survey, 1983). Duerk was successful in accurately measuring discharge by the constant-injection method for rates of change in stage of as much as 5 ft/hr (550 ft³/s/hr) on a recession. While limited, his data showed greater errors for measurements on the rising-stage portion of hydrographs. Constant-injection dilution measurements by the senior author in a lined canal in Colorado indicated good accuracy on a recession having a rate of change of stage of 1.6 ft/hr (360 ft³/s/hr) but with poor results on a rising hydrograph: -10 percent on the average for a 5.5 ft/hr rate of change (940 ft³/s/hr).

Automatic dilution gaging

There has always been a need to measure flows and develop ratings for small streams. The need has increased as interest has increased in land-use impact resulting, for example, from urban development and coal mining. Flows from small basins are characteristically unsteady with rapidly changing stage hydrographs of 1 to 6 hours duration and turbulent, high-velocity debris and sediment-laden flows. Such conditions make conventional measurement by current meter virtually impossible. Remoteness of the small basins and the unpredictability of flow events further complicate conventional measuring techniques, even if measuring such streams were physically possible.

The application of dilution gaging to the automatic measurement of runoff hydrographs involves the marriage of any one of the types of injection apparatus previously discussed with some sort of water-sampling system located at the gage site. The system is activated by a

stage or discharge indicative of storm runoff. Such a system could be installed at a station until rated and then moved to other stations as needed.

Injection system

Figure 30A shows a recommended injection system with these features:

1. A constant-head mariotte tank has been coupled with a 12-volt constant-rate injection pump, thus sealing the dye reservoir from dirt and evaporation and improving pump performance by ensuring constancy of injection rate.
2. A small graduated section (see mariotte vessel on the right in fig. 14) has been incorporated into the top of the larger mariotte reservoir to permit checking the flow rate each time the station is visited.⁴ It is important that an accurate measurement of the injection rate be obtained and that it reflect the resistance from the plumbing of the injection apparatus. This is especially true of a multiple-injection scheme. A dye solution of the same concentration should be added to the tank after each rate check, keeping the tank full.
3. A valve and flushing-tank arrangement is incorporated into the system to allow cleansing of the pump and injection lines each time the system is visited. An alcohol solution should be used as a flushing agent to cleanse, prevent freezing, and discourage insects from entering the injection lines.
4. The injection may be initiated by a stage sensor at the injection site or by hard-wire relay from the sampling station. Stage sensors should be set to initiate and stop injection above and below a given stage.
5. The dye reservoir should be adequate for a continuous injection throughout the expected duration of the hydrograph.

⁴If there is sufficient flow at the time of a routine visit, the flow-rate check might also be made a routine dye-dilution discharge measurement, thus lending confidence to future automatic measurements.

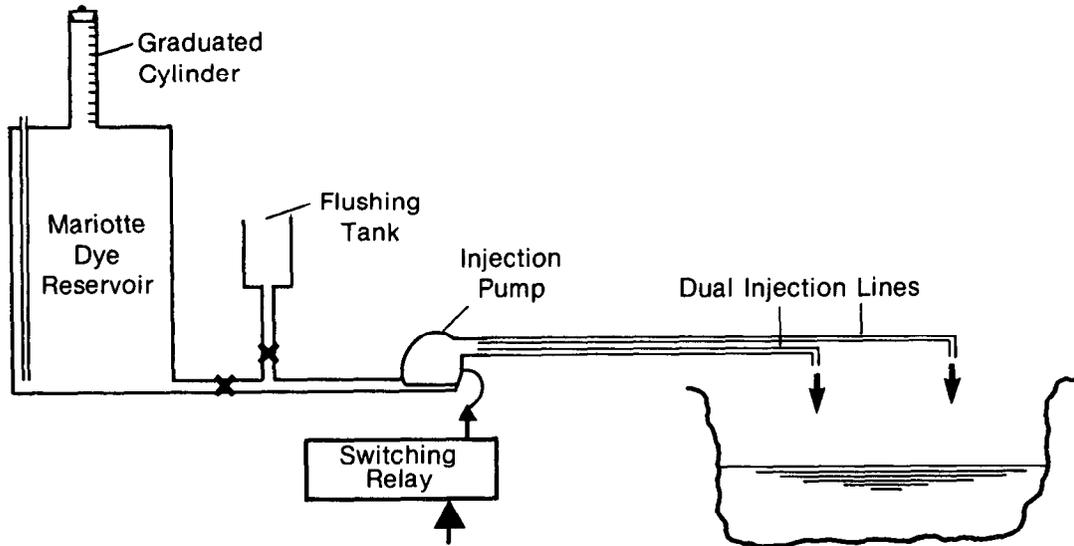
6. The dye solution should contain alcohol if freezing conditions are expected. The concentration of the solution and the pump rate should be sufficient to yield concentrations of about 5 $\mu\text{g/L}$ using rhodamine WT 20 percent (see fig. 9) at the maximum expected discharge.
7. Unless the channel reach above the gaging station where sampling is to take place is narrow and the flow likely to be highly turbulent, a two-point injection is recommended to minimize the necessary mixing length. As can be seen from the section on Mixing Length, a two-point injection shortens mixing length by a factor of four, compared to single-point injection.

Sample recovery system

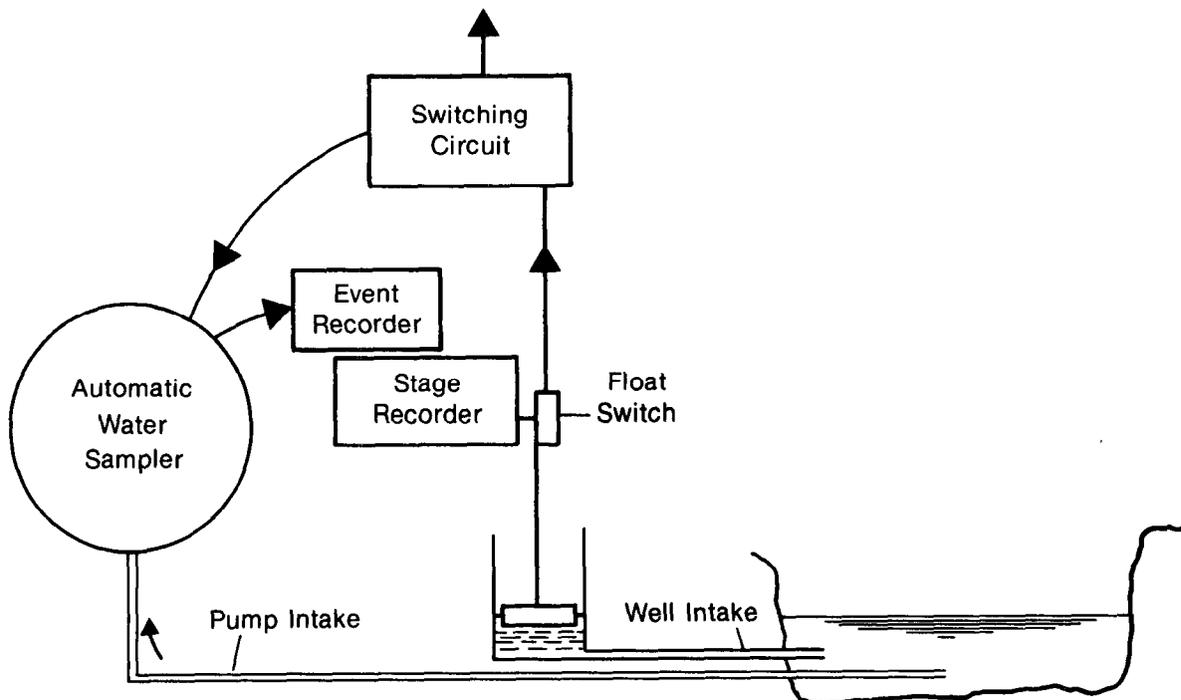
Several commercial, bottle-type water samplers are available and may be incorporated into a recovery system. The recommended sampling system is shown in figure 30B and should have these features (Duerk, 1983):

1. It must be capable of obtaining a sample when flow velocities are high, preferably from an intake located well within the main flow.
2. The sampler must take discrete samples and not allow cross-contamination between samples. For highly responsive streams, the shortest pumping time necessary to obtain a sample should be used so that each sample more closely represents an exact stage and discharge.
3. Individual samples must be coordinated with stage and time.
4. As with all dilution-discharge measurements, background samples are needed. This may be done by having the sampler turn on simultaneously with the dye-injection system so that a sample is obtained before the dye arrives.

The technique suggested in (4) above would not allow evaluation of background during the dye-injection period, and the assumption would have to be made that once the samples had been permitted to set in the laboratory, background was constant. If rhodamine WT dyes or



A. Automatic Injection System



B. Automatic Sample-Recovery System

Figure 30.—Automatic injection and sampling systems for dye-dilution measurement of unsteady flow.

pontacyl pink are used, that is probably a safe assumption. If other tracers are used, variation in background with flow may be a problem. The collection of samples upstream from the injection system at selected stages during the flow event would provide evidence of any background variation.

The biggest deterrent to the accurate dye-dilution measurement of storm-produced runoff flows is possible error due to dye losses in flows transporting large sediment loads, particularly fine clay sediments. That source of error can be partly countered by using high dye concentrations where permissible and possibly by developing a means of filtering the samples immediately upon collection.

Velocity Area Measurements

One of the earliest forms of discharge measurement was the "salt-velocity" method (Allen, 1927). It consists of measuring the time of passage of a pulse of sodium chloride in a full-flowing conduit of uniform and known cross-sectional area and length. Using that technique, the discharge may be computed as

$$Q = \frac{V}{t_c} \quad (13)$$

where V is the volume of the conduit through which the lapsed time of travel of the tracer, t_c , is measured.

That method is still entirely feasible and very accurate for discharge measurement using a variety of tracers. Tracer losses no longer become a factor, the only essential requirements being a single uniform-flow cross section throughout the test reach and a steady flow. Fluorescent dyes are well suited to such measurements (Harris and Sanderson, 1968).

The following precautions should be taken:

1. Lapsed time, T_c , should be based on measurements of the centroids of upstream and downstream time-concentration curves. Lapsed time should not be based on time

from slug injection to downstream time-concentration curves unless unavoidable or the distance between observation points is very large.

2. Careful synchronization of the time between observation points is essential.
3. Physical dimensions of the conduit need to be checked if feasible; as-built dimensions may be slightly different from blueprint dimensions. A slug-injection type dye-dilution discharge computation may also be made, using the same data as for the velocity test. Then conduit geometry would not be a factor in the accuracy of the measurement. Assuming that mixing length is adequate and care is exercised in performing the test, as discussed previously, the discharge by dilution should confirm the results of the velocity test (using the same data) if the volume of the conduit is correct.

Selected Bibliography

- Aastad, J., and Reinhardt, S., 1954, Discharge measurements by means of a salt solution, the relative dilution method: International Union of Geodesy and Geophysics, International Association of Scientific Hydrology, General Assembly of Rome, v. 3, no. 38, p. 289-292.
- Allen, C. M., 1927, Hydraulic-turbine tests by the Allen method: *Power Plant Engineering*, 31, no. 10, p. 549-551.
- Duerk, M. D., 1983, Automatic dilution gaging of rapidly varying flow, U.S. Geological Survey Water-Resources Investigations 83-4088.
- Dunn, B., and Vaupel, D. E., 1965, Effects of sample and fluorometer-compartment temperatures on fluorometer readings, in *Geological Survey Research 1965: U.S. Geological Survey Professional Paper 525-D*, p. D225-227.
- Fischer, H. B., and others, 1979, *Mixing in inland and coastal waters*: New York, Academic Press, Inc., 483 p.
- Frederick, B. J., 1964, Measurement of turbine discharge with radioisotopes: U.S. Geological Survey Report TEI-855, 32 p.
- Guy, H. P., and Norman, V. W., 1970, Field methods for measurement of fluvial sediment: U.S. Geological Survey Techniques of Water-Resources Investigations, Book 3, Chapter C2, 59 p.
- Harris, D. D., and Sanderson, R. B., 1968, Use of dye tracers to collect hydrologic data in Oregon: *American Water Resources Association Bulletin*, v. 4, no. 2, p. 51-68.
- Holley, E. R., Dilution method of discharge measurement

- in pipes: Flow Measurement Symposium, National Bureau of Standards, Gaithersburg, Maryland, October 1977, Special Publication 484, Proceedings p. 395-421.
- Katz, B. G., and Fisher, G. T., 1983, A comparison of selected methods for measuring flow rate in a circular storm sewer: 1983 International Symposium on Urban Hydrology, Hydraulics, and Sediment Control, University of Kentucky, Lexington, Kentucky, July 25-28, p. 359-369.
- Kilpatrick, F. A., 1968, Flow calibration by dye-dilution measurement: *Civil Engineering*, February, p. 74-76.
- _____ 1969, A comprehensive evaluation of dye-dilution discharge measurement applications: ASCE Hydraulics Division 17th Annual Specialty Conference, Utah State University, Logan, Utah, August 20, 1969.
- _____ 1970, Dosage requirements for slug injection of rhodamine BA and WT dyes, in *Geological Survey Research: U.S. Geological Survey Professional Paper 700-B*, p. B250-253.
- Morgan, W. H., Kempf, D., and Phillips, R. E., 1977, Validation of use of dye dilution method for flow measurement in large open and closed channel flows: Flow Measurement Symposium, National Bureau of Standards, Gaithersburg, Maryland, February 23-25, 1977, 34 p.
- Ostrem, G., 1964, A method of measuring water discharge in turbulent streams: *Geographical Bulletin*, no. 21, May 1964 (Publication of the Geographical Branch, Ottawa, Canada).
- Schuster, J. C., and Hansen, R. L., 1968, Discharge measurements using radioisotopes in high-head turbines and pumps at flatiron power and pumping plant: Colorado-Big Thompson Project, TID-25177, Bureau of Reclamation Report no. 40, Hydraulic Branch, Chemical Engineering Branch, November 1968, Bureau of Reclamation, Denver, Colorado, 116 p.
- Smart, P. L., and Laidlaw, I. M. S., 1977, An evaluation of some fluorescent dyes for water tracing: *Water Resources Research*, February, v. 13, no. 1, p. 15-33.
- Spencer, E. A., and Tudhope, J. R., 1958, A literature survey of the salt-dilution method of flow measurement: *Institute of Water Engineers Journal*, v. 12, p. 127-138.
- Wilson, J. F., Jr., Cobb, E. D., and Kilpatrick, F. A., 1984, Fluorometric procedures for dye tracing: U. S. Geological Survey Techniques of Water-Resources Investigations, Book 3, Chapter A12 (in press).
- Yotsukura, N., and Cobb, E. D., 1972, Transverse diffusion of solutes in natural streams: U.S. Geological Survey Professional Paper 582-C, 19 p.
- Yotsukura, N., and Kilpatrick, F. A., 1973, Tracer simulation of soluble waste concentration: *American Society of Civil Engineers, Journal of Environmental Engineering Division*, v. 99, no. EE4, p. 499-515.

APPENDIX A

COMPUTATION OF PERCENTAGE MIXING

Computation of Percentage Mixing

The degree of mixing of a tracer is a measure of the extent to which mixing has occurred laterally in a channel cross section. Thus, when a tracer is uniformly mixed across a channel, the degree of mixing is 100 percent. From a theoretical standpoint, 100-percent mixing requires an infinite channel length, but for practical purposes, mixing on the order of 95 percent is ordinarily sufficient to yield an accurate discharge measurement. When greater accuracy is required, mixing on the order of 98 percent may be desirable.

Ordinarily, the computation of percentage mixing is not necessary, because inspection of the concentration data will indicate the adequacy of mixing. Furthermore, if those data are discharge weighted, less than ideal mixing is acceptable. Several mixing equations exist; the one by Yotsukura and Cobb (1972, p. C5) is recommended. This may be expressed as

$$P_m = 100 - \frac{50}{\bar{c}Q} \sum_{i=1}^N |c_i - \bar{c}| Q_i \quad (14)$$

where c_i are the concentrations at points i across the section or, if the slug injection is used, the areas under the concentration-time curves as meas-

ured at those same points. Fluorometer dial readings adjusted for background may be used in lieu of concentrations if all are on the same scale;

\bar{c} is the average of the plateau concentrations, c_i , as determined by discharge weighting or by arithmetic averaging; dial readings adjusted for background may also be substituted if all are on one scale;

Q_i are the discharges applicable to the points, i ; and

Q is the total stream discharge and must equal the sum of the Q_i 's.

The term $|c_i - \bar{c}|$ results in absolute values.

An example computation of percent mixing follows, using the data from the example described in figure 18.

$$\begin{aligned} P_m &= 100 - [|32.0 - 44.4| 3 + |42.0 - 44.4| 5 + |58.4 \\ &\quad - 44.4| 11 + |40.0 - 44.4| 17 + |37.2 - 44.4| 4] \\ &\quad \frac{50}{44.4 \times 40} \\ &= P_m = 100 - [|12.4| 3 + |2.4| 5 + |14.0| 11 \\ &\quad + |4.4| 17 + |7.2| 4] 0.0282 \\ &= 91.3 \text{ percent.} \end{aligned}$$

APPENDIX B

FORMS AND SAMPLE COMPUTATIONS

Constant-rate dye-dilution discharge measurement on _____ Date _____
 G.H. = _____; Est. Q = _____; Required injection rate _____ mL/min
 C = _____
 Est. mixing length required, _____ ft. Injection started @ _____ stopped @ _____; measured q = _____ mL/min

FIELD DATA			FLUOROMETRIC ANALYSIS				DATA ANALYSIS AND COMPUTATIONS						
Sampling Locations	Bottle no.	Lapsed time, min	Estimated portion of total discharge, Q_h (ft ³ /s)	Fluorometer scale			Conc. in $\mu\text{g/L}$	Weighted conc., columns 10 x 5	Weighted avg. conc. reading $\bar{c} = \frac{\sum C_i q_i}{Q_{est}}$	Computed discharge $Q = 5.89 \times 10^{-7} q \bar{c}$			
Distance	Pt	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Stream backgr.													
Stream @	a												
ft	b												
downstr.	c												
	d												
	e												
Σ													
Stream @	a												
ft	b												
downstr.	c												
	d												
	e												
Σ													
Stream @	a												
ft	b												
downstr.	c												
	d												
	e												
Σ													

FLUOROMETER CALIBRATION						
Standard	Concentration in $\mu\text{g/L}$	1X	3X	10X	30X	Net dial reading, R_D
Standard						
Dist. Water						

(a) Enter arithmetic average if weighting is not used.
 (b) Use to define fluorometer calibration for scale used with stream samples.

Figure 31.—Form for recording, analyzing, and computing constant-rate dye-dilution discharge measurement using actual concentrations.

Constant-rate dye-dilution discharge measurement on _____ Date _____ / _____ / _____
 G.H. = _____ @ _____ ; Estimated $Q =$ _____ ; Required injection rate _____ mL/min *
 C = _____ ; Corresponding dilution factor, DC relative to $C_s =$ _____
 Estimated mixing length required, _____ ft. Injection started @ _____ Injection stopped @ _____

FIELD DATA				FLUOROMETRIC ANALYSIS					DATA ANALYSIS AND COMPUTATIONS				
Sampling Locations	Distance	Pt	Bottle no.	Lapsed time, min	Estimated portion of total discharge, Q_n (ft ³ /s)	Fluorometer scale				Net dial reading, \bar{r}	Weighted dial readings, columns 10 x 5	Weighted avg. dial reading $\bar{r} = \frac{\sum Col 11}{Q_{est}}$	Computed discharge $Q = 5.89 \times 10^{-7} \frac{R}{\bar{r}} \times \frac{D_c}{D_T}$
						1X	3X	10X	30X				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	
Stream backgr.													
Stream @	a												
ft	b												
downstr.	c												
	d												
	e												
Σ													
Stream @	a												
ft	b												
downstr.	c												
	d												
	e												
Σ													
FLUOROMETER CALIBRATION													
Standard	Concentration in $\mu\text{g/L}$												
Standard	1X	3X	10X	30X	Net dial reading, R								Total dil. factor, D_T
Dist. Water													

* $D_c=1.0$ if D_T is based on C rather than C_s .

Figure 33.—Form for recording, analyzing, and computing constant-rate dye-dilution discharge measurement using fluorometer dial readings and dilution factors.

